Cooperative Relaying with Low-Density Lattice Coding and Joint Iterative Decoding

Bin Chen, Dushantha N. K. Jayakody and Mark F. Flanagan

School of Electrical, Electronic and Communication Engineering
University College Dublin
Dublin, Ireland

ISTC 2014, Bremen
August 22, 2014
Background

- Diversity techniques offer an effect method to against fading by providing different versions of data.
Background

- Diversity techniques offer an effect method to against fading by providing different versions of data.
- Cooperation has been proposed to provide diversity gain, called cooperative diversity.
  - Relay channel (classical single-source single-relay model)
  - User cooperation (multiple-user model with one or more relays)
  - MIMO system
Background

- Diversity techniques offer an effect method to against fading by providing different versions of data.
- Cooperation has been proposed to provide diversity gain, called cooperative diversity.
  - Relay channel (classical single-source single-relay model)
  - User cooperation (multiple-user model with one or more relays)
  - MIMO system
- Coded Cooperation: distributed channel coding (distributed Turbo/LDPC codes) can provide both coding gain and diversity gain.
Diversity techniques offer an effective method to combat fading by providing different versions of data.

Cooperation has been proposed to provide diversity gain, called *cooperative diversity*.

- Relay channel (classical single-source single-relay model)
- User cooperation (multiple-user model with one or more relays)
- MIMO system

Coded Cooperation: distributed channel coding (distributed Turbo/LDPC codes) can provide both coding gain and diversity gain.

*Compute-and-Forward* is a method to compute and reliably transmit linear functions of transmitted messages over the wireless channel.

Research has focused on proving theoretical potential of this technique; practical designs have yet to be created.
Diversity techniques offer an effect method to against fading by providing different versions of data.

Cooperation has been proposed to provide diversity gain, called cooperative diversity.

- Relay channel (classical single-source single-relay model)
- User cooperation (multiple-user model with one or more relays)
- MIMO system

Coded Cooperation: distributed channel coding (distributed Turbo/LDPC codes) can provide both coding gain and diversity gain.

Compute-and-Forward is a method to compute and reliably transmit linear functions of transmitted messages over the wireless channel.

- Research has focused on proving theoretical potential of this technique; practical designs have yet to be created.

Lattice codes are a natural fit for compute-and-forward in wireless channels.

- The same real algebra underlies both encoder and channel.
Random codes not good for compute-and-forward

**Random codes**
- Sum of codewords is **not** a codeword.
- Must decode individual messages.

**Structured codes**
- Sum of codewords is a codeword.
- Can decode **integer combinations** of messages.
Low Density Lattice Codes

Theoretical basis

- With proper shaping and lattice decoding, lattice codes can approach Poltyrev's capacity, and can also approach the AWGN capacity at any SNR [Erez and Zamir, 04].
- Low density lattice codes (LDLCs) were introduced in [Sommer et al., 08], where it was shown that LDLCs can perform close to capacity with low decoding complexity.

Encoding:

- Lattice codeword: \( x = Gb \).
- \( b \) is an integer information vector and \( G \) is an \( n \times n \) lattice generator matrix.

Decoding:

- An iterative scheme for calculating the PDF \( f(x_k | y) \).
- Message passing algorithm is implemented between variable nodes and check nodes.
- Make decision based on the estimates of the the final variable node PDFs.
Most existing work on coded cooperation focuses on LDPC and turbo codes.

- We focus on structured codes (LDLCs).
- Propose a scheme for LDLC cooperative transmission in Multiple Access Relay Channel (MARC) uplink.
Scope of presented work

- Most existing work on coded cooperation focuses on LDPC and turbo codes.
  - We focus on structured codes (LDLCs).
  - Propose a scheme for LDLC cooperative transmission in Multiple Access Relay Channel (MARC) uplink.
- Present two relaying methods, one having low decoding complexity and one having a greater power efficiency.
Most existing work on coded cooperation focuses on LDPC and turbo codes.

- We focus on structured codes (LDLCs).
- Propose a scheme for LDLC cooperative transmission in Multiple Access Relay Channel (MARC) uplink.

Present two relaying methods, one having low decoding complexity and one having a greater power efficiency.

Propose an efficient joint iterative decoding structure and algorithm.


<table>
<thead>
<tr>
<th>Transmission Protocol</th>
<th>Transmission Structure</th>
<th>Receiver Structure</th>
<th>Simulation Results</th>
<th>Summary</th>
</tr>
</thead>
</table>

## Three-Phase Transmission Model

![Diagram of Three-Phase Transmission Model]

- Consider a Multiple Access Relay Channel (MARC) where User 1 and User 2 transmit their own information packets to the same destination via a shared relay.
- The relay node decodes and forwards a combination message to the destination.
- Three transmission phases (half duplex).
Three-Phase Transmission Model

- **Message:** Integer (L-PAM) message sequences $b_1 \in \{0, 1, 2, \ldots, L - 1\}^N$ and $b_2 \in \{0, 1, 2, \ldots, L - 1\}^N$
- **Shaping:** Map each integer information vector $b_i$ to another integer vector $b'_i$, so each element of the lattice codeword is uniformly distributed over $[-L/2, L/2)$
- **Encoding:** Form the LDLC codeword $x'_i = Gb'_i$, which lies in the lattice $\Lambda$
- **Linear structure:** $b_i = b'_i \mod L$ and therefore $x'_i = x_i \mod \Lambda$
Three-Phase Transmission Model

First two phases \((i = 1, 2)\):

\[
\begin{align*}
\mathbf{y}_i^R &= \sqrt{P_i} h_i^R \mathbf{x}'_i + \mathbf{n}_i^R, \\
\mathbf{y}_i^D &= \sqrt{P_i} h_i^D \mathbf{x}'_i + \mathbf{n}_i^D,
\end{align*}
\]
Three-Phase Transmission Model

Decoding at relay:

\[ \hat{x}_i = \text{LDLCdecoder} \left( \frac{y_i^R}{h_i^R \sqrt{P_i}} \right), \quad i = 1, 2 \]
- **Superposition LDLC (S-LDLC)**: In this method, we simply add the lattice codewords to form

\[ x'_3 = \hat{x}_1 + \hat{x}_2 \]

which is equivalent to addition of the underlying information vectors, since

\[ x'_3 = \hat{x}_1 + \hat{x}_2 = G \cdot (\hat{b}_1 + \hat{b}_2) \]

- **Modulo-Addition LDLC (MA-LDLC)**: In this method, in order to improve the power efficiency of the relay, the LDLC codewords can be added modulo the lattice to form

\[ x'_3 = [\hat{x}_1 + \hat{x}_2] \mod \Lambda = G \cdot b'_3 \]

where

\[ b'_3 = \hat{b}_1 + \hat{b}_2 \mod L \]
**Superposition LDLC (S-LDLC):** In this method, we simply add the lattice codewords to form
\[ \mathbf{x}'_3 = \hat{\mathbf{x}}_1 + \hat{\mathbf{x}}_2 \]
which is equivalent to addition of the underlying information vectors, since
\[ \mathbf{x}'_3 = \hat{\mathbf{x}}_1 + \hat{\mathbf{x}}_2 = \mathbf{G} \cdot (\mathbf{b}_1 + \mathbf{b}_2) \]

**Modulo-Addition LDLC (MA-LDLC):** In this method, in order to improve the power efficiency of the relay, the LDLC codewords can be added modulo the lattice to form
\[ \mathbf{x}'_3 = [\hat{\mathbf{x}}_1 + \hat{\mathbf{x}}_2] \mod \Lambda = \mathbf{G} \cdot \mathbf{b}'_3 \]
where
\[ \mathbf{b}'_3 = \hat{\mathbf{b}}_1 + \hat{\mathbf{b}}_2 \mod L \]

MA-LDLC provides better power efficiency, but requires more complex decoding at destination.
The lattice point $x'_3$, which can be considered as a network coded component, will be transmitted from the relay to the destination, i.e.,

$$y^D_3 = \sqrt{P_3} h^D_3 x'_3 + n^D_3$$

Finally, the destination uses the signals $y^D_1$, $y^D_2$ and $y^D_3$ to jointly decode $b_1$ and $b_2$. 

Bin Chen, Dushantha N. K. Jayakody and Mark F. Flanagan

Cooperative Relaying with Low-Density Lattice Coding and Joint Iterative Decoding
Three-Phase Transmission Model

- **Key problem**: how to efficiently recover the information symbols at the destination based on the signals received.

- Therefore, we next focus on the joint iterative decoding structure.
At destination, the LDLC decoders receive three packets from three independent channels.
- **Inner iterations**: each LDLC decoder performs $M$ inner iterations.
**Outer iteration**: After every $M$ inner iterations of LDLC decoding, the network coding nodes implement one outer iteration to exchange extrinsic information between the LDLC decoders.
**Outer iteration:** The extrinsic information $R^a(x)$ will be considered as *a priori* information for each decoder in the next iteration.
**Final decision**: after several inner and outer iterations, the final variable node messages are calculated to make decisions for information symbol vector $b_i$. 
Joint Decoding Algorithm

Factor graph
**Joint Decoding Algorithm**

- **Decoder** $i$: initialize each variable node $x_{i,k}$ to send the message

$$f_{i,k,j}^{(0)}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_{i,k}^D / (h_i^D \sqrt{P_i}) - x_{i,k})^2}{2\sigma^2}}$$

to neighboring check node $c_{i,j}$
- **Inner iterations**: each LDLC decoder performs $M$ inner iterations.
  
  1. Check-to-variable messages
  2. Variable-to-check messages
Joint Decoding Algorithm

**Check node message at decoder i & iteration t:** each check node $c_{i,j}$ sends a PDF function $Q_{i,j,k}^{(t)}(x)$ in $t$-th iteration to each of its neighboring variable nodes $x_{i,k}$.

a. **Convolution step:** All messages, except $f_{i,k,j}(x)$, are convolved. For ease of illustration, we assume that the $d$ non-zero values in $j$-th row of matrix $H$, $\{h_1, h_2..., h_d\}$, lie in the first $d$ columns.

\[
\tilde{p}_{jk}(x) = f_{i,1,j}(\frac{x}{h_1}) \otimes \cdots f_{i,k-1,j}(\frac{x}{h_{k-1}}) \\
\otimes f_{i,k+1,j}(\frac{x}{h_{k+1}}) \otimes \cdots \otimes f_{i,d,j}(\frac{x}{h_d})
\]

b. **Stretching step:**

\[
p_{i,j,k}(x) = \tilde{p}_{i,j,k}(-h_kx)
\]

c. **Periodic extension step:** extend with period $1/|h_k|

\[
Q_{i,j,k}^{(t)}(x) = \sum_{a=-\infty}^{\infty} p_{i,j,k}\left(x - \frac{a}{h_k}\right)
\]
Variable node message at decoder $i$ & iteration $t$: each variable node $x_{i,k}$ sends a message $f_{i,k,j}^{(t)}(x)$ in $t$-th iteration to each of its neighboring check nodes $c_{i,j}$.

a. Variable node update rule:

$$\tilde{f}_{i,k,j}^{(t)} = e^{-\frac{(y_{i,k} - x_{i,k})^2}{2\sigma^2}} R_{i,k}^{(t)}(x) \prod_{l=1}^{n} Q_{i,l,k}^{(t)}(x)$$

b. Normalization step:

$$f_{i,k,j}^{(t)}(x) = \frac{\tilde{f}_{i,k,j}^{(t)}(x)}{\int_{-\infty}^{\infty} \tilde{f}_{i,k,j}^{(t)}(x) dx}$$
**Outer iteration:** After $M$ inner iterations, each variable node $x_{i,k}$ will send the extrinsic message $R^{e}_{i,k}(x)$ to the corresponding network coding node $n_k$:

$$R^{e}_{i,k}(x) = e^{-\frac{(y_{i,k} - x_{i,k})^2}{2\sigma^2}} \prod_{l=1}^{n} Q^{(M)}_{i,l,k}(x)$$
S-LDLC:

\[ R_{1,k}^e(x') \downarrow \]

convolution

\[ \otimes \]

\[ R_{3,k}^2(x') \]

\[ R_{2,k}^e(x') \uparrow \]
Joint Decoding Algorithm

MA-LDLC:

\[
R_{1,k}^e(x') \xrightarrow{H} R_{1,k}^e(b') \xrightarrow{\text{mod}} R_{1,k}^e(b) \xrightarrow{\text{convolution}} \]

\[
\bigotimes \xrightarrow{\text{mod}} R_{3,k}^a(b') \xrightarrow{\text{shaping}} R_{3,k}^a(x')
\]

\[
R_{2,k}^e(x') \xrightarrow{H} R_{2,k}^e(b') \xrightarrow{\text{mod}} R_{2,k}^e(b) \xrightarrow{\uparrow}
\]
The resulting messages $R_{i,k}^a(x)$ are generated from the network coding nodes. Each LDLC decoder will repeat the inner iterative decoding and the outer iterations until the maximum iteration number is achieved.
Final decision: after \((M \cdot N)\) iterations, the final variable node message is calculated via

\[
\tilde{f}_{i,k,j}^{(M \cdot N)} = e^{-\frac{(y_{i,k} - x_{i,k})^2}{2\sigma^2}} R_{i,k}^a(x) \prod_{l=1}^{n} Q_{i,l,k}^{(M \cdot N)}(x).
\]
Then, the integer vector $\mathbf{b}_i$ is estimated as

$$\hat{x}_{i,k} = \arg \max \tilde{f}_{i,k,j}^{(M \cdot N)}(x), \quad i = 1, 2$$

and

$$\hat{b}_i = \lfloor H\hat{x}_i \rfloor \mod L, \quad i = 1, 2$$
Simulation scenario

We compare S-LDLC and MA-LDLC with three competing schemes in a slow Rayleigh fading environment:

- Non-cooperative LDLC
- Network coded 4-PAM cooperation without channel coding (NC)
- Network-Turbo-coded 4-PAM cooperation (NTC) [C. et al., 14]

We set the link SNRs as follows:

- $\gamma_{S1D} = \gamma_{S2D}$
- $\gamma_{SR} = \gamma_{RD} = \gamma_{S1D} + 6$dB

All compared schemes were normalized to have the same overall transmitted power.
We compare S-LDLC and MA-LDLC with other schemes in the scenario of non-perfect SR-links.

- Cooperative schemes outperform non-cooperative schemes.
- The proposed MA-LDLC has better performance than the reference schemes (6.2dB gain over NC, 2.5dB gain over NTC 4-PAM).
For fixed block length, MA-LDLC outperforms S-LDLC.

Further coding gain can be provided as the block length increases.
Outage probability predicts better outage performance (high diversity) for the cooperative schemes compared to non-cooperative scheme.

Outage analysis provide a bound on the FER performance; the proposed scheme performs closer to this bound as the block length increases.
Summary

- We have proposed a new scheme for coded cooperation, based on distributed low-density lattice codes.

- Outlined two approaches for relay processing: S-LDLC and MA-LDLC:
  - S-LDLC has low decoding complexity due to the simple superposition of codewords.
  - MA-LDLC has better performance (shaping gain) due to the usage of modulo-addition which can improve the power efficiency of the relay.

- Designed an efficient joint iterative decoding structure at the destination node.

- Proposed scheme provides 2.5dB SER gain over network-turbo-coded 4-PAM and may be easily extended to the case of multiple sources.
Thank You!
Latin square LDLC

Every row and column of the parity matrix $\mathbf{H}$ has the same $d$ nonzero values, except for a possible change of order and random signs. The sorted sequence of these $d$ values $h_1 \geq h_2 \geq \ldots \geq h_d \geq 0$.

*Example* ($6 \times 6$):

\[
\begin{pmatrix}
0 & -h_2 & 0 & -h_3 & h_1 & 0 \\
h_2 & 0 & 0 & h_1 & 0 & -h_3 \\
0 & h_3 & h_1 & 0 & h_2 & 0 \\
0 & 0 & -h_3 & -h_2 & 0 & h_1 \\
h_1 & 0 & 0 & 0 & h_3 & h_2 \\
h_3 & -h_1 & h_2 & 0 & 0 & 0
\end{pmatrix}
\]
Low triangular parity matrix

In order to be more convenient for encoding and shaping, [Sommer el al.08] proposed a simpler structure for $H$, a low-triangular parity matrix. The column degree of the rightmost column of $H$ will start from 1 and gradually increase until $d$. In the same manner, the row degree of the top row will be 1, and it will gradually increase until $d$.

Example ($8 \times 8$):

$$
\begin{pmatrix}
  h_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & h_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  h_2 & 0 & h_1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & -h_2 & h_1 & 0 & 0 & 0 & 0 \\
  -h_3 & 0 & 0 & h_2 & h_1 & 0 & 0 & 0 \\
  0 & -h_2 & 0 & h_3 & 0 & h_1 & 0 & 0 \\
  0 & -h_3 & 0 & 0 & h_2 & 0 & h_1 & 0 \\
  0 & 0 & -h_3 & 0 & 0 & h_2 & 0 & h_1
\end{pmatrix}
$$
Iterative decoding algorithm for LDPC and LDLC

Common points:

- The iterative algorithm is represented by using a message passing scheme over the bipartite graph of the code.

Difference:

- LDPC: the messages are scalar values (LLR).
- LDLC: the messages are real functions over interval $(-\infty, \infty)$.
- LDPC: each iteration the check nodes send messages to the variable nodes along the edges of the bipartite graph and vice versa.
- LDLC: each iteration the check nodes send message are periodic extensions of pdfs. The messages sent by the variable nodes are pdfs.

Unlike LDPC-based schemes, in LDLC both the encoder and the channel use the same real algebra which is natural for the continuous-valued channel.
Low-density lattice codes

- Low-density lattice codes (LDLC) can achieved the capacity of AWGN channels.
- Defined by a real-valued sparse parity check matrix, LDLC codes are analogue to finite alphabet LDPC codes.
- With linear-time iterative BP decoder, LDLC codes outperform LDPC coded modulation based on multilevel coding and non-binary LDPC codes.
- Compared with non-binary LDPC codes, LDLC codes are more natural for the continuous-valued channels.
- LDLC codes introduce no redundancy to original information data since the parity check matrix are Latin square matrix.
- Practical decoding algorithms in which passing messages are approximated as Gaussian mixtures, largely reduce the computation and storage complexity in decoding.
Symbol error rate (SER) for various block lengths.

- 100
- 1000
- 10,000
- 100,000

distance from capacity [dB]

0 0.5 1.5 2 2.5 3 3.5 4
Introduction

Transmission Structure

Receiver Structure

Simulation Results

Summary

Bin Chen, Dushantha N. K. Jayakody and Mark F. Flanagan

School of EECE, University College Dublin, Ireland

Cooperative Relaying with Low-Density Lattice Coding and Joint Iterative Decoding

channel PDF

check node message #1

check node message #2

check node message #3

check node message #4

Final variable node message
Random codes not good for computation