Generalized concatenated MIMO system with GMD decoding

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Outline

MIMO Systems and Space-Time Codes

Generalised Concatenated Codes

Product Codes

Simulation

Conclusion
There are three important existing components in this work that were not used together before:

- Generalised concatenated codes with inner Space-time Block Codes.
- Product codes.
- Generalised minimum distance decoding.
Problem Solved [2/2]

To make them work together the following problems had to be solved:

- Find a good reliability metric for space-time block codes that doesn’t require additional computations.
- Create a generalised minimum distance decoder for product codes.
- Design a code system that combines the above components and can achieve frame error rate $10^{-7}$ at 13 dB. Length should be about 16000 bit.
We will only consider perfect receiver channel state information (perfect R-CSI).

\[ r_m = \sum_{n=1}^{N} \alpha_{n,m} s_n + \eta_m \]

\( \alpha_{n,m} \) are i.i.d. complex Gaussian random variables. This channel is called a Rayleigh channel.
Space-Time Block Codes

\[ \mathbf{r} = \mathbf{H} \cdot \mathbf{M} \cdot \text{diag}(m_0 \ldots m_0) \cdot \left( \mathbf{s} + (m_1 \ldots m_1)^T \right) + \eta \]

\[ m_0 = \sqrt{\frac{6}{M^2 - 1}} = \sqrt{\frac{2}{5}} \quad m_1 = -\frac{M - 1}{2} = -\frac{3}{2} \]

Space-time block code can be thought of as a rotation in \( \mathbb{R}^8 \). They can also be regarded as a lattice in \( \mathbb{R}^8 \) or \( \mathbb{C}^4 \). Golden codes are space-time codes with 4 complex information symbols, length 2 for two transmit antennas and at least two receive antennas. Golden codes are constructed as a canonical embedding of an ideal of a quaternion algebra into a complex space.
Embedded Space-Time Codes

This way we divide Golden code over QAM-16 into 256 coset of cardinality 256. This subcode is known to have better determinant distance.
Metric $d_2 - d_1$ can be interpreted as the noise energy needed to change the decoder decision.
Reliability metric

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Reliability metric performance

- Decoded successfully
- Decoded with errors
Design of Generalised Concatenated Code

- We have an embedded system of two Golden codes and two outer codes.
- The information symbols of first Golden code are coded with first outer code.
- The information symbols of second Golden code are coded with second outer code.
- Both outer codes are defined over $GF(2^8)$.
- A mapping between information symbols of outer codes and information symbols of Golden code is denoted as $\mathcal{L}$.
**Decoder Design**

1. Decode first inner codes.
2. Decode first outer code.
3. Decode second inner codes with corrections from step 2.
4. Decode second outer code.
A codeword of a product code is a matrix whose rows are codewords of the first component code and whose columns are codewords of the second component code.
1. Decode all column codes in-place.
2. Decode all row codes in-place.
3. If at least one component code corrected errors, repeat.
1. First we will try to decode the outer code without erasures.
2. If it fails, we will erase two least reliable symbols in each column.
3. Repeat previous step until the word is successfully decoded or we make $d_c$ erasures.
Proposed code

- Code length is 2048 time samples.
- The first outer code is a product of two $[32, 24, 9]_{2^8}$ Reed-Solomon codes.
- The second outer code is a product of $[32, 30, 3]_{2^8}$ and $[32, 28, 5]_{2^8}$ Reed-Solomon codes.
- The inner codes are Golden codes with QAM-16 modulation. They are partitioned into $2^8$ cosets of $2^8$ cardinality.
- The rates of outer codes are 0.56 and 0.82.
- The rate of the proposed code system is 0.69 symbols (over $GF(2^8)$) per time sample. It can also be interpreted as rate 0.69 relative to the rate of the inner code.
Eb\textsubscript{N0}, dB

Frame error rate

Without GMD
With GMD
Improved

Inner Code 1
Inner Code 2
Reed-Solomon code
The lower bound
In this work we have proposed new generalised concatenated coded modulation for MIMO systems. The outer code is a product code, for which we use iterative decoding algorithm. The inner codes are embedded Golden codes. The proposed GMD decoder has energy gain of 0.2–0.4 dB. The decoder for proposed generalised concatenated system has reasonable complexity. The computer simulation showed that this code system reaches frame error rate of $10^{-7}$ at signal to noise ratio 13.2 dB per bit.
Thank you for your attention!