# Error Localization Using Erasure Decoding

CSE Seminar
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# Overview

- Coding Theory
- Erasure decoding
- Reed Solomon
- CM vs. DM
- CM, DM and Erasure Decoding

# Coding Theory

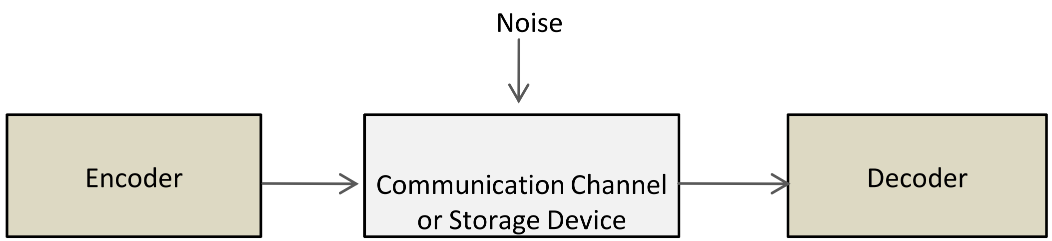


**Fig 1.** Typical system for encoding and decoding data

| Information symbols | Parity symbols |
|---|---|
| k | n-k=2t |

**Fig 2.** Codeword

- Robust mechanism for checking and correcting errors needed.
- ***Hamming*** distance – number of symbols any two codewords are separated by.
- Common measure of the error-correcting capability of a system is the ***minimum Hamming distance*** between valid codewords ($d_{\min}$)
- Channel errors can occur at randomly isolated locations or in finite length sequences (burst errors)
- A linear block code with minimum distance $d_{\min}$ can correct up to $\left\lfloor \frac{d_{\min}-1}{2} \right\rfloor$ symbol errors.
- In some cases it is possible to determine the positions of corrupted symbols prior to decoding

# Interleaving

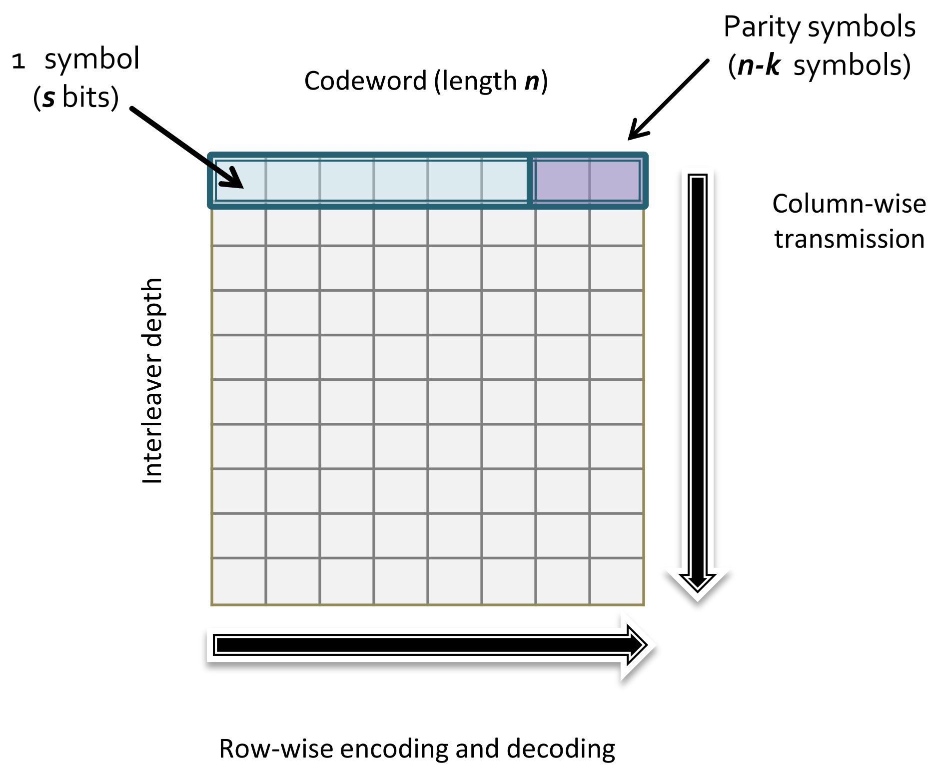


**Fig.** 3 10-bit interleaved system

A burst of symbol errors is distributed among multiple received words and fewer errors occur in each received codeword.

A burst error across 12 symbols is spread such that it causes a maximum of 2 symbol errors in each received codeword.

A codeword in the block is correctly decoded if the number of errors in it is smaller than ***t***.

Very likely that received words in one interleaver block have the same error locations.

# Interleaving

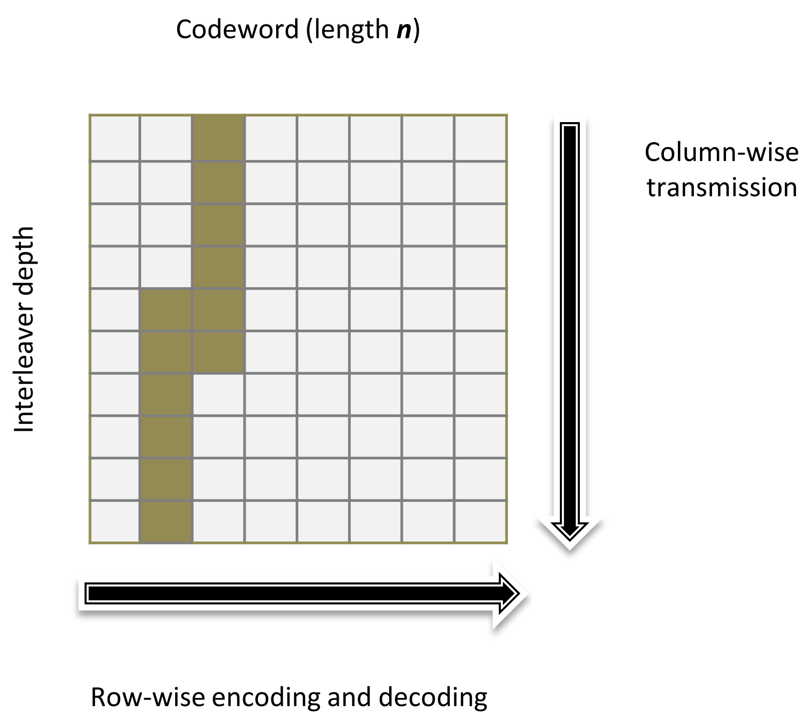


**Fig. 4** 10-bit interleaved system with burst errors

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# Erasure decoding

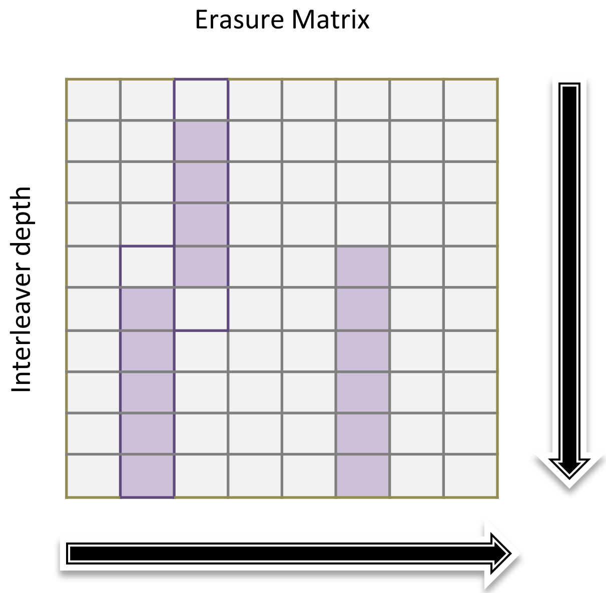


**Fig. 5** 10-bit interleaved system – erasure matrix

Use the correlation between the error positions of consecutive codewords to inform the decoder.

A symbol with known error position but unknown error value is called *an erasure.*

Decode one codeword (perhaps better protected) by using standard methods, flag error positions as erasures in adjacent codewords.

Decode the rest of the codewords in the interleaved block using error-and-erasure decoding.

# Erasure decoding

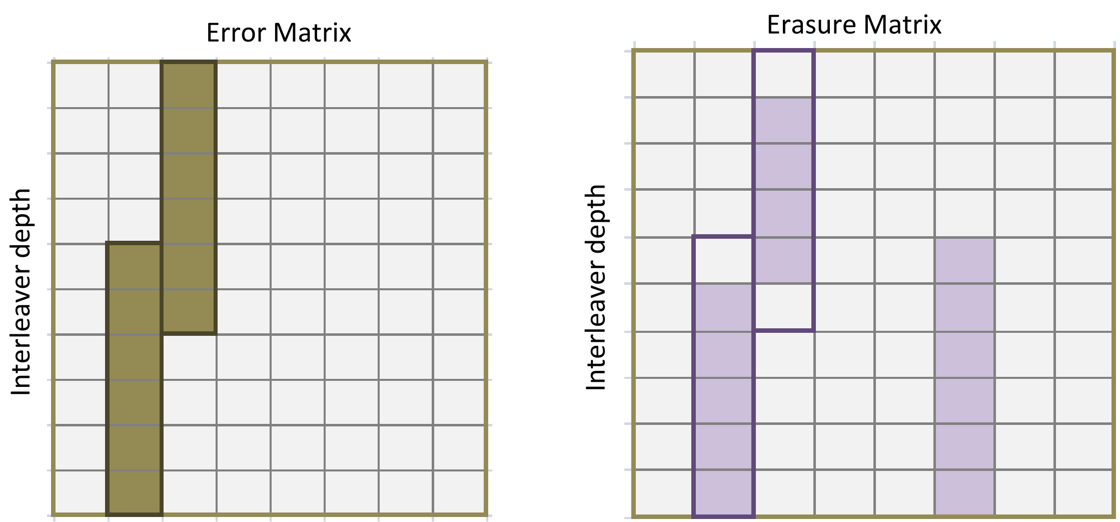


Fig. 6 10-bit interleaved system – both error and erasure matrices

Given an ***n***-symbol codeword with ***k*** data symbols, an ***(n,k)*** linear block can correct ***e*** errors and ***v*** erasures provided that:

$$2e + v \le d_{\min} - 1 \le n - k = 2t$$

# Erasure decoding

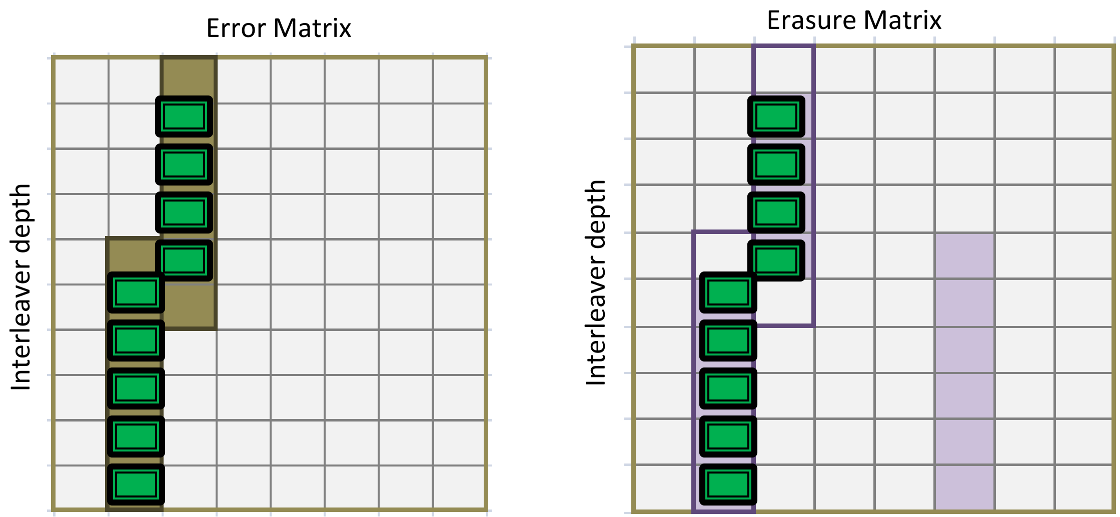


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**Fig. 6** 10-bit interleaved system – both error and erasure matrices

# Reed Solomon Encoder

- General structure of an RS encoder:

$$c(x) = g(x)i(x)$$

where ***c(x)*** is the codeword, ***i(x)*** is the information and ***g(x)*** is a ***generator polynomial*** of the form:

$$g(x) = (x - a^{i})(x - a^{i+1})\ldots(x - a)^{i+2t-1}$$

- The ***2t parity symbols*** in a systematic RS codeword are given by:

$$p(x) = i(x)x^{n-k} \bmod g(x)$$

- The ***received codeword*** is given by:

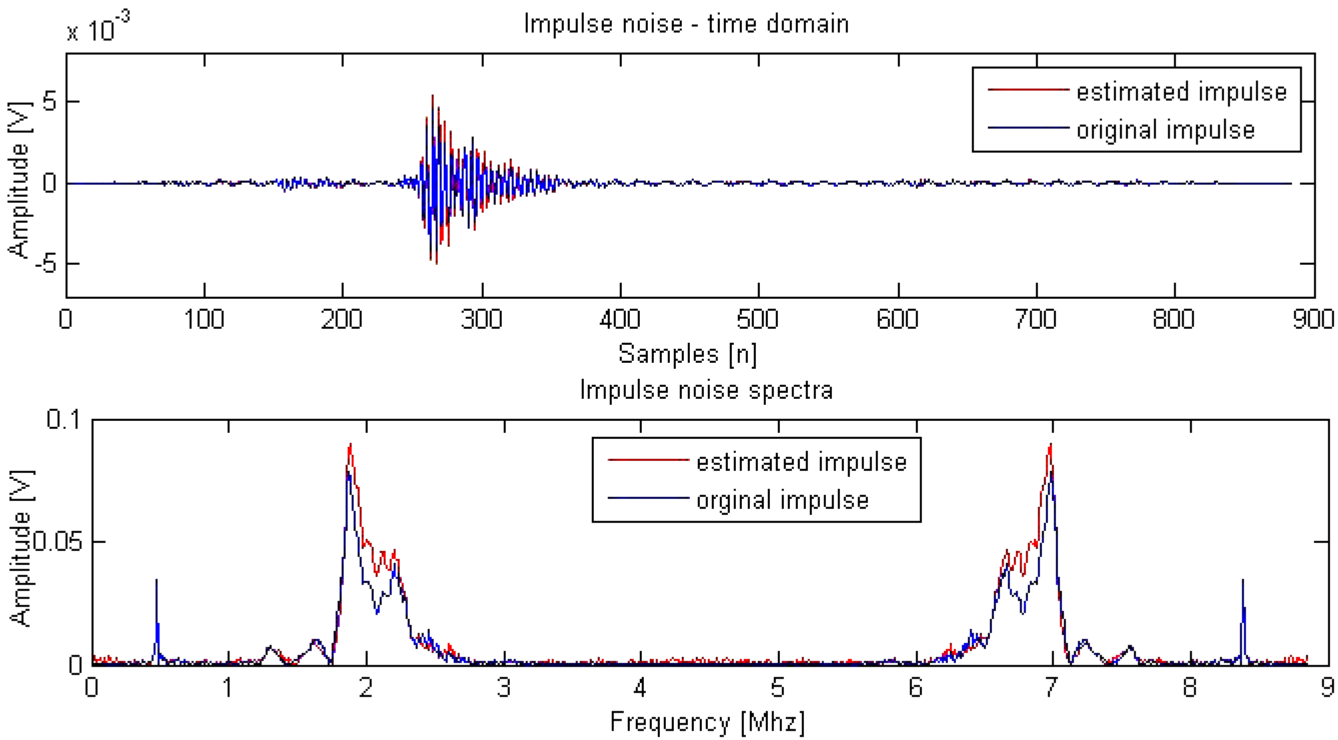
$$r(x) = c(x) + e(x)$$

# Reed Solomon

- Compute syndromes associated with each received word.
- Compute an error-locator polynomial for one received codeword.
- Use the polynomial as an erasure polynomial.
- Compute the error-locator polynomials for the remaining codewords.
- Compute error magnitudes and perform error correction.

- A ***syndrome*** is a string of digits representing parity checks on a series of bits in symbols or series of symbols.
- In the case of RS, it can be calculated by substituting the ***2t*** roots of the generator polynomial into ***r(x)***, which results in ***2t*** syndroms of ***s*** bits each.
- Indicate whether the block of codewords has errors, and if case of errors, get error locations and error values.
- Typical RS architecture : syndrome calculator, error locator, error magnitude calculator, error corrector.
- High coding gain at a high code rate (k/n)
- Wide range of applications: wireless or mobile communications (cellular telephones, microwave links, satellite communications, digital TV, high speed modems , DSL, CDs, DVDs, etc. )

# Impulse Noise Estimation




Transfer function estimated using 10,000 DM and CM measurements.

$$T_{xy}(f) = \frac{P_{yx}(f)}{P_{xx}(f)}$$

# CM and erasures

- Impulse noise, burst erasures. Use block interleaving to make bursts look like random errors.
- Use the CM signal as a reference to flag erasures for RS decoding.

**Context:**

- DMT (Discrete Multitone) simulation , DSL system with impulse noise and AWGN (-120dBm/Hz).
- Since high correlation between CM and DM is present, estimate impulse noise present in DM.
- Threshold it, FFT it, flag the carriers affected by impulse noise , compute erasure matrix.
- No need to implement RS decoder in our case, count.
- Compute statistics based on transmitted codewords, erasure and error matrices.
- Results sensitive to threshold chosen for flagging erasures. Appropriate threshold levels have been determined by simulations.

# Simulations

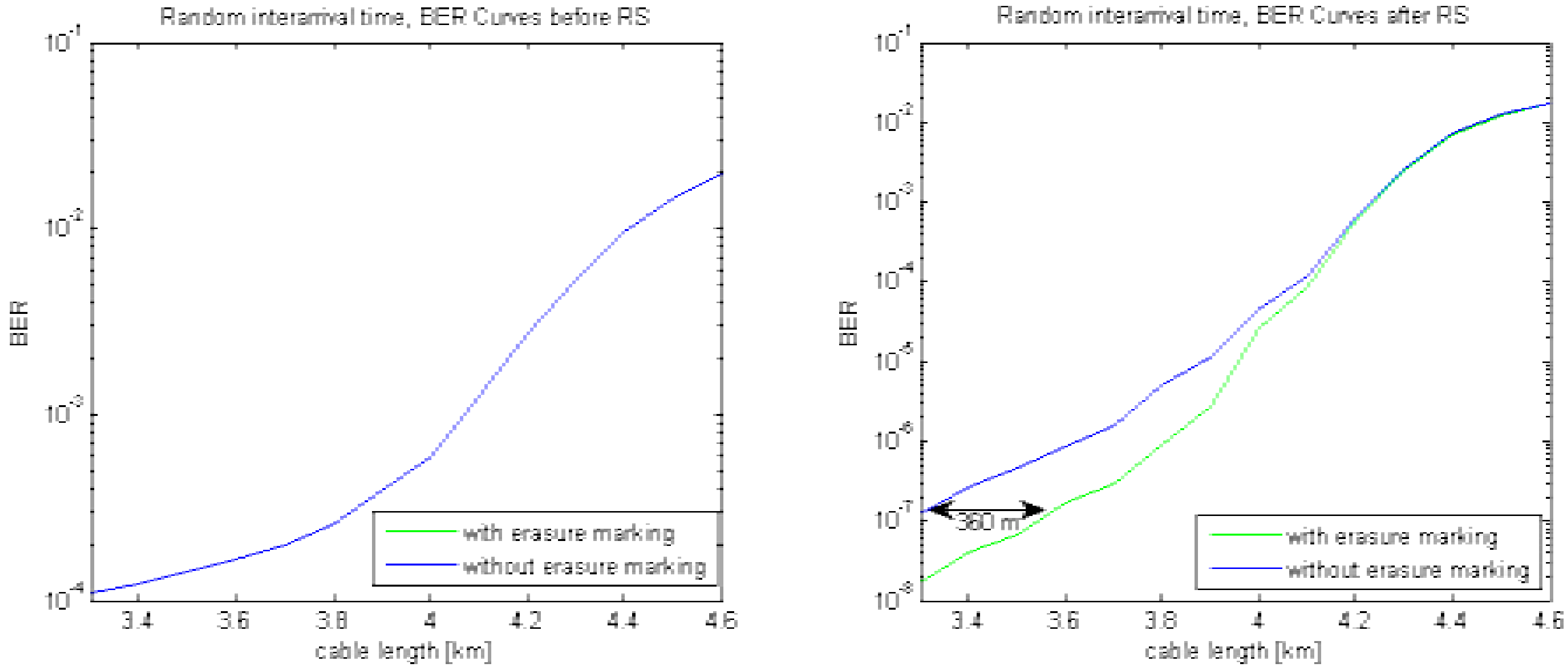



**Fig. 14** BER after RS decoding in a DSL system

# Thank you! Questions?