MIMO Systems in the Subscriber-Line Network

Georg Tauböck and Werner Henkel
Telecommunications Research Center Vienna (FTW)
E-mail: tauboeck@ftw.at Werner.Henkel@ieee.org

Abstract— **We consider multi-tone modulation (DMT, Discrete Multitone, OFDM, Orthogonal Frequency Division Multiplex) over cable bundles and present a compact mathematical description of the transmission line. Both, near- and far-end crosstalk (NEXT, FEXT) are taken into account and a new method for removing far-end crosstalk is derived.**

Keywords— **MIMO, Multiple-Input-Multiple-Output, FEXT-Cancellation, NEXT-Cancellation, DMT, Discrete Multitone, xDSL**

I. Multi-Input/Multi-Output Systems and their Mathematical Description

In this paper, we consider a transmission system, which consists of several loops from one dedicated point to another (e.g., from the central office to the cabinet). We assume that these loops are used for bidirectional data transmission. Let K denote the number of loops which are used for transmission from point A to point B and L the number of loops which are used in the opposite direction (from B to A). This MIMO channel can be described using a matrix $(h_{ij})_{i=1..K,j=1..K}$ and a matrix $(g_{nm})_{n=1..K,m=1..L}$ of time-discrete impulse responses. Let a_j and b_m denote the transmitted symbol sequences from point A to point B on loop j and from point B to point A on loop m. Then the received symbol sequence c_k on loop k is given by (c.f., Fig. 1)

$$c_k = h_{kk} * a_k + \underbrace{\sum_{j=1, j\neq k}^{K} h_{kj} * a_j}_{\text{FEXT}} + \underbrace{\sum_{m=1}^{L} g_{km} * b_m}_{\text{NEXT}} . \quad (1)$$

For a moment, however, let us concentrate on only one loop with a DMT transmission over it. The data is divided into blocks of length N and each block (conjugate-complex extended) is passed through an IFFT. Some redundancy is added, i.e., a guard interval of length ν is prepended; through a transmit filter and the channel, the data reaches the receiver, where after a receive filter and further processing, the redundancy is eliminated. An FFT and a frequency domain equalization (which simply performs scalar multiplications with complex factors) is carried out and a decision device recovers the original data. As shown in [1], many of the above operations can be described using matrices[1]:

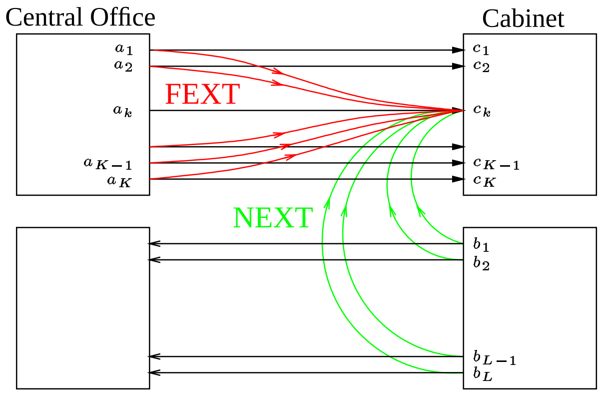

Fig. 1. FEXT and NEXT in a MIMO system without self-NEXT; self-FEXT would mean the b_i and c_i to be collocated.

$$\underline{z} = \underline{Z} \cdot \underline{C}_R \cdot \underline{F} \cdot \underline{G}_R \cdot \underline{H} \cdot \underline{G}_I \cdot \underline{F}^{-1} \cdot \mathcal{C}_I(\underline{x}) \quad (2)$$

Here, $\underline{x}$ denotes the N-dimensional symbol vector to be transmitted, $\underline{z}$ the N-dimensional symbol vector at the input of the decision device, $\mathcal{C}_I$ the non-linear operation of conjugate complex extension (it transforms an N-dimensional vector into a $2N$-dimensional vector) and $\underline{F}$ the $(2N) \times (2N)$ DFT matrix; the $(2N+\nu) \times (2N)$ matrix $\underline{G}_I$ describes the addition of the guard interval, the $(2N+\nu) \times (2N+\nu)$ matrix[2] H represents the channel impulse response (note that transmit filter, receive filter, and further processing are considered as part of the channel), the $(2N) \times (2N+\nu)$ matrix $\underline{G}_R$ removes the guard interval, the $N \times (2N)$ matrix $\underline{C}_R$ removes the second N components of a $2N$-dimensional vector (inversion of the conjugate complex extension), and the diagonal $N \times N$ matrix $\underline{Z}$ performs frequency-domain equalization. It is also assumed that ν is chosen greater than the length of the channel impulse response (i.e., where it significantly differs from zero). Then it can be shown [1] that $\underline{G}_R \cdot \underline{H} \cdot \underline{G}_I$ represents a cyclic convolution and therefore $\underline{F} \cdot \underline{G}_R \cdot \underline{H} \cdot \underline{G}_I \cdot \underline{F}^{-1}$ is a $(2N) \times (2N)$ diagonal matrix. Furthermore, it can be easily seen that

[1] This equation describes the transmission of one block which is part of a sequence of transmitted blocks

[2] $\underline{H}$ is a convolution matrix: it consists of the elements of the channel impulse response

$\underline{C}_R \cdot \underline{F} \cdot \underline{G}_R \cdot \underline{H} \cdot \underline{G}_I \cdot \underline{F}^{-1} \cdot \mathcal{C}_I(\underline{x}) = \underline{S} \cdot \underline{x}$ with an $N \times N$ diagonal matrix $\underline{S}$.

Now, we return to our MIMO system with a DMT modulation scheme on each loop, all having the same parameters (especially, N and ν). We further neglect NEXT, taking resort to various techniques to remove it (e.g., frequency division for the multiplexing of upstream and downstream data). Applying[3] $\underline{a}_k = \underline{G}_I \cdot \underline{F}^{-1} \cdot \mathcal{C}_I(\underline{x}_k)$ and $\underline{y}_k = \underline{C}_R \cdot \underline{F} \cdot \underline{G}_R \cdot \underline{c}_k$ to (1) and using the linearity of the operations, we get[4]

$$\begin{aligned} \underline{y}_k &= \underline{C}_R \cdot \underline{F} \cdot \underline{G}_R \cdot \underline{H}_{kk} \cdot \underline{G}_I \cdot \underline{F}^{-1} \cdot \mathcal{C}_I(\underline{x}_k) + \\ &+ \sum_{j=1, j\neq k}^{K} \underline{C}_R \cdot \underline{F} \cdot \underline{G}_R \cdot \underline{H}_{kj} \cdot \underline{G}_I \cdot \underline{F}^{-1} \cdot \mathcal{C}_I(\underline{x}_j) \\ &= \sum_{j=1}^{K} \underline{S}_{kj} \cdot \underline{x}_j . \end{aligned} \quad (3)$$

Let $y_k(n)$ and $x_j(n)$ denote the n-th component (the n-th carrier) of the N-dimensional vectors $\underline{y}_k$ and $\underline{x}_j$, respectively, and $s_{kj}(n)$ the n-th diagonal element of the $N \times N$ diagonal matrix $\underline{S}_{kj}$. Then (3) can be equivalently written as

$$y_k(n) = \sum_{j=1}^{K} s_{kj}(n) \cdot x_j(n), \quad n = 1, \ldots, N \quad (4)$$

or in a more compact way as matrix-vector multiplications

$$\overline{y}(n) = \overline{A}(n) \cdot \overline{x}(n), \quad n = 1, \ldots, N \quad (5)$$

with

$$\begin{aligned} \overline{y}(n) &:= (y_1(n) \cdots y_K(n))^T , \\ \overline{A}(n) &:= (s_{kj}(n))_{k=1..K, j=1..K} , \text{ and} \\ \overline{x}(n) &:= (x_1(n) \cdots x_K(n))^T . \end{aligned}$$

Note that overlined letters specify matrices and vectors that contain transmit functions and signals at one frequency, only, for all loops, whereas underlining specifies matrices and vectors with multiple frequencies at one loop.

[3]In fact, we do not only transmit one block but a sequence of blocks, but under certain assumptions (all impulse responses h_{kj}, $j = 1, \ldots, K$, have approximately the same delay and their lengths are smaller than the length of the guard interval ν) there is no inter-block interference and it suffices to consider only one block

[4]$\underline{y}_k$ denotes the symbol vector before frequency-domain equalization of the k-th loop

II. Equalization of DMT-MIMO Systems

In this section, we derive a new DMT-MIMO system equalization method, or, to formulate it in another way, we present a method for removing FEXT.

First of all, observe that there does **not** occur FEXT if and only if the *system matrices* $\overline{A}(n)$ are diagonal for all $n = 1, \ldots, N$. This becomes obvious from comparing Eqn. (5) with Eqn. (1). Some processing is necessary to obtain such diagonal system matrices.

Up to now, there exist proposals to remove FEXT in the literature [1], which are based on the inversion of the matrices $\overline{A}(n)$, either in the transmitter or in the receiver (this leads to the desired diagonal unit matrix). However, these methods require that all matrices $\overline{A}(n)$ are invertible, which is not guaranteed in general. Bridge taps, e.g., lead to zeros in the transmit function at some frequencies. Inversion may then be problematic. Furthermore, inversion at the transmitter may cause stronger disturbances of loops outside the MIMO bundle and inversion at the receiver may enhance noise.

We, instead, follow another strategy of computing singular value decompositions (SVDs) [2] of the matrices $\overline{A}(n)$, i.e.[5],

$$\overline{A}(n) = \overline{Q}(n) \cdot \overline{\Lambda}(n) \cdot \overline{P}^H(n), \quad n = 1, \ldots, N \quad (6)$$

with unitary[6] $K \times K$ matrices $\overline{Q}(n)$ and $\overline{P}(n)$ and real diagonal $K \times K$ matrices $\overline{\Lambda}(n)$, whose elements are greater than or equal to zero.

Let $\overline{t}(n)$ denote the K-dimensional symbol vector to be transmitted on the n-th carrier over the whole cable bundle. We perform multiplications of these vectors with the matrices $\overline{P}(n)$, the resulting symbol vectors $\overline{x}(n) := \overline{P}(n) \cdot \overline{t}(n)$ are used as input of the DMT-MIMO system and the output symbol vectors $\overline{y}(n) = \overline{A}(n) \cdot \overline{x}(n)$ are then multiplied with the matrices $\overline{Q}^H(n)$, which yields symbol vectors

$$\begin{aligned} \overline{r}(n) &:= \overline{Q}^H(n) \cdot \overline{y}(n) \\ &= \overline{Q}^H(n) \cdot \overline{A}(n) \cdot \overline{x}(n) \\ &= \overline{Q}^H(n) \cdot \overline{A}(n) \cdot \overline{P}(n) \cdot \overline{t}(n) \\ &= \overline{Q}^H(n) \cdot \overline{Q}(n) \cdot \overline{\Lambda}(n) \cdot \overline{P}^H(n) \cdot \overline{P}(n) \cdot \overline{t}(n) \\ &= \overline{\Lambda}(n) \cdot \overline{t}(n) , \end{aligned} \quad (7)$$

where we used Eqn. (6) and the property $\overline{U}^{-1} = \overline{U}^H$ of a unitary matrix $\overline{U}$. Note that the resulting system matrices are the diagonal matrices $\overline{\Lambda}(n)$, eliminating FEXT

[5]$\overline{P}^H(n)$ denotes the complex conjugated and transposed (also called Hermitian transposed) matrix $\overline{P}(n)$

[6]also called orthonormal

completely. After frequency domain equalization (for all loops and frequencies, separately), we finally obtain the transmitted symbol vectors $\bar{t}(n)$.

Now we are able to state the **algorithm** for removing FEXT. The following steps have to be carried out:

1. only once in the startup-phase, as long as the transmission properties do not change:
   (a) determination of the system matrices $\overline{A}(n)$;
   (b) calculation of the SVDs of the matrices $\overline{A}(n)$, i.e., $\overline{A}(n) = \overline{Q}(n) \cdot \overline{\Lambda}(n) \cdot \overline{P}^H(n)$;
2. during transmission, for each symbol block of the cable bundle:
   (a) in the transmitter multiplications of the symbol vectors to be transmitted ($\bar{t}(n)$) with the matrices $\overline{P}(n)$, i.e., $\overline{x}(n) := \overline{P}(n) \cdot \bar{t}(n)$;
   (b) transmission of the resulting symbol vectors ($\overline{x}(n)$) over the DMT-MIMO system, i.e., $\overline{y}(n) = \overline{A}(n) \cdot \overline{x}(n)$;
   (c) in the receiver multiplications of the received symbol vectors ($\overline{y}(n)$) with the matrices $\overline{Q}^H(n)$, i.e., $\overline{r}(n) := \overline{Q}^H(n) \cdot \bar{t}(n)$;
   (d) frequency domain equalization (for all loops and frequencies, separately).

Remarks: Compared to other proposals, this method has the advantage that a singular value decomposition of a matrix is always well defined. Furthermore, because of the unitarity of the matrices $\overline{P}(n)$ and $\overline{Q}^H(n)$, the average power of all loops stays the same and there is neither an increased disturbance of other loops nor an average noise enhancement. We also want to emphasize that this concept for FEXT removal can be extended using a two-dimensional (over frequencies and loops) bit-loading and power-distribution algorithm and additional coding in space (loops) / time / frequency may be applied, too (c.f., *space-time coding* [3]).

III. Conclusions

We presented a general description of symbol transmission over cable bundles including FEXT and NEXT. We applied it to a DMT transmission scheme; the result was a compact mathematical formulation which was based on simple matrix-vector multiplications. Singular value decompositions of the system matrices were the key to equalize the FEXT channel. Finally, we motivated that two-dimensional bit-loading can be applied and code constructions similar to so-called space-time codes may be implemented.

References

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