UEP MIMO-OFDM with Beamforming-Combining for Imperfect Channel Information

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Abstract— **In this paper, unequal error protection (UEP) adaptive multiple-input multiple-output (MIMO) system using orthogonal frequency division multiplexing (OFDM) is considered. Our proposed transmission scheme consists of three stages. The inner stage is a UEP adaptive loading, which is introduced to fulfill a certain UEP profile by distributing bits and power across the unitarily transformed eigenbeams. The next stage is a variable size beamforming to adaptively utilize the channel eigenbeams. The final stage is a post combiner that uses a linear-spatial equalizer to withstand the rapid wireless channel variation and resolve the undesired CSI error effects. For the case of imperfect channel state information (CSI), we studied the performance of different beamforming-combining techniques for time varying channel conditions and different correlation models using an intuitive and a robust bit-loading. The results are promising and fulfill the proposed UEP profile. Moreover, the robust bit-loading seems to be a reliable technique in case of CSI uncertainties.**

I. INTRODUCTION

FREQUENCY selective MIMO channels suffer from serious performance deterioration. However, this could be resolved by cutting down the whole bandwidth into smaller subchannels. OFDM techniques are used in conjunction with the existing MIMO systems to transfer such selective channels into N flat subbands. Additionally, the MIMO-OFDM combination takes advantages of simple equalization and channel adaptation capabilities.

In general, wireless systems with feedback does not extremely improve the spectral efficiency [8]. Nevertheless, the system performance is significantly improved when the CSI is known at the transmitter side (CSIT), such that the transmission can be adapted to the channel variations. To achieve an optimum performance, the channel coefficients would need to be known *accurately* at the transmitter. However, a perfect CSIT knowledge is a rather impractical assumption due to estimation errors, limited feedback constraint, channel feedback delay, and/or quantization errors. Therefore, a partial CSI at the transmitter is a more realistic assumption between the two extremes, full-CSI and no-CSI [5]. Currently, there are two main partial feedback schemes: the channel mean feedback $\overline{\mathbf{H}}$ [5] and the channel correlation feedback $\mathbf{R_H} = E\{\mathbf{H}^{\mathrm{H}}\mathbf{H}\}$ [6].

Adaptively, bits and power are allocated according to the eigenbeams of the spatial channel between the transmit and the receive antennas as in [3]. Ideally, in MIMO systems, singular-value decomposition (SVD) can maintain the eigenbeam orthogonality in case of accurate CSI knowledge. However, this orthogonality is highly threatened if there is a considerable mismatch between the CSIT, the instantaneous channel, and CSI at the receiver (CSIR). Furthermore, the eigenvalues vary with the channel spatial correlation variations.

To compensate for the orthogonality distortion and the inter-eigenmode interferences due to channel variations, we consider the problem of maximum SNR beamforming-combining [8]. In this method, the beamforming weights (pre-combiner) and the spatial equalizer (post-combiner) are employed at the transmitter using N_T antennas and the receiver using N_R antennas, respectively [7],[5]. However, the transmitter has only access to the limited feedback CSI, which could be different from the exact CSI.

Consequently, different eigenvalues offer the chance for UEP realizations despite of CSI errors [2]. Hence, a certain UEP profile is fulfilled by distributing bits and power [2] across the variable eigenbeam levels. Therefore, we propose a UEP bit and power loading based on a modified version of the non-UEP bit-loading by Chow, Cioffi, and Bingham, as in [1]. This UEP loading algorithm devotes different numbers of bits and power levels onto the realized eigenmodes with different priorities [2]. Furthermore, optimized beamforming-combining is considered to minimize the inter-eigenmode interference for different spatial correlation scenarios.

The robust bit-loading scheme proposed in [1] is considered to overcome unexpected interferences.

The rest of the paper is organized as follows: Section 2 describes the adaptation principles in MIMO-OFDM with partial CSI. Section 3 states the proposed UEP loading and beamforming. Section 4 discusses some simulation results. Finally, the paper is concluded with Section 5.

II. Adaptive MIMO-OFDM

MIMO techniques can ideally provide independent channels just like OFDM when SVD is used for diagonalization. Singular values (eigenbeams), together with OFDM subchannels, can be used for bit and power allocation across the spatial and the spectral domain. This changes the bit and power loading to be a two-dimensional (frequency and space) problem. In this work, the proposed UEP bit-loading algorithm in [1] is adapted to the MIMO-OFDM two dimensions by allocating bits and power across the eigenbeams for each frequency according to a required noise margins, i.e, UEP requirements.

A. Channel Model

We consider a MIMO channel matrix $\tilde{\mathbf{H}}_k$ for each subchannel k. Assuming that the channel matrix has a complex Gaussian distribution, where $\tilde{\mathbf{H}}_k \in \mathcal{C}\,\mathcal{N}(\overline{\tilde{\mathbf{H}}}_k, N_T\sigma^2_{\tilde{\mathbf{H}}}\mathbf{I})$. The statistical mean and the variance can be estimated at the receiver.

The transmit and receive antennas are exposed to a limited scattering environment that results in a non-identity correlation matrix. Where the transmitter correlation and the receiver correlation matrices are $\mathbf{R}_{\mathrm{tx}}$ and $\mathbf{R}_{\mathrm{rx}}$, respectively. The equivalent channel matrix $\mathbf{H}_k$ is defined as [9]

$$\mathbf{H}_k \equiv \mathbf{R}_{\mathrm{rx}}^{1/2} \cdot \tilde{\mathbf{H}}_k \cdot \mathbf{R}_{\mathrm{tx}}^{1/2} \ . \tag{1}$$

B. Limited CSI Regime

Partial CSI is achieved by transmitting either:

- **the channel statistical mean** $\overline{\mathbf{H}}_k$, (the mean of $\mathbf{H}_k$) that could be estimated channel at the receiver in case of a frequency division duplexing (FDD) and at the transmitter in case of a time division duplexing (TDD), or
- **the channel correlation matrix**, $\mathbf{R}_{\mathbf{H}} = E\{\mathbf{H}^{\mathrm{H}}\mathbf{H}\}$, where the power is allocated to the directions of the maximum eigenvalues without any frequency dependency.

Throughout this paper, the channel statistical mean $\overline{\mathbf{H}}_k$ estimated at the receiver is fed back to the transmitter through a delayed feedback channel. Within the feedback duration, the CSIT is considered to be deterministic even if the channel changed in between. This certainly leads to a CSI uncertainty.

C. System Analysis

The SVD of $\overline{\mathbf{H}}_k = \overline{\mathbf{U}}_{\mathbf{k}}\overline{\mathbf{D}}_{\mathbf{k}}\overline{\mathbf{V}}_{\mathbf{k}}^{\mathrm{H}}$, where $\overline{\mathbf{V}}_k$ and $\overline{\mathbf{U}}_k$ are unitary matrices and $\overline{\mathbf{D}}_k$ is a diagonal matrix with the singular values for the k^{th} subcarrier. As in [5] and [7], the optimum beamforming can be maintained at the transmitter by knowing $\overline{\mathbf{V}}_k$ or only L columns of $\overline{\mathbf{V}}_k$. The CSI error model is adopted from [5] as $\hat{\mathbf{H}}_k = \overline{\mathbf{H}}_k + \Xi_k$, where $\hat{\mathbf{H}}_k$ is the instantaneous channel model and the estimated CSI at the receiver (assuming a perfect CSI at the receiver side). $\Xi_k \sim \mathcal{CN}(0, \sigma^2_{\Xi})$ represents the CSI error. Therefore, the received vector $\mathbf{Y}_k$ of the k^{th} subchannel can be written as

$$\begin{aligned} \mathbf{Y}_k &= \hat{\mathbf{H}}_k\overline{\mathbf{V}}_k\mathbf{P}^{1/2}\mathbf{X}_k + n_k \\ &= \overbrace{\hat{\mathbf{U}}_k\hat{\mathbf{D}}_k\hat{\mathbf{V}}_k^{\mathrm{H}}\overline{\mathbf{V}}_k\mathbf{P}^{1/2}}^{\mathbf{\Psi}_k}\mathbf{X}_k + na_k \ , \end{aligned} \tag{2}$$

where $\mathbf{X}_k$ is the transmitted vector, $n_k \sim \mathcal{CN}(0, \sigma_n^2)$ is the additive white Gaussian noise vector, and $\mathbf{\Psi}_k$ represents the aggregated channel matrix. $\mathbf{P}^{1/2}$ is the allocated power, which is also part of $\mathbf{\Psi}_k$. The estimation of $\mathbf{\Psi}_k$ is easier than the estimation of $\hat{\mathbf{H}}_k$, due to the reduced off-diagonal elements [7]. Moreover, there is no need for an extra signaling to inform about $\mathbf{P}^{1/2}$, as it could be estimated inherently as a part of $\mathbf{\Psi}_k$ [7].

The post-combiner at the receiver is just a linear equalizer, which can be a zero-forcing (ZF) or a linear minimum mean square error (MMSE) techniques. Hence, the equalizer output is $\mathbf{Z}_k = \mathbf{W}_k\mathbf{Y}_k$. The weighting matrix for ZF is

$$\mathbf{W} = \{\mathbf{\Psi}^{\mathrm{H}}\mathbf{\Psi}\}^{-\mathbf{1}}\mathbf{\Psi}^{\mathrm{H}} \ , \tag{3}$$

and for MMSE is

$$\mathbf{W} = \{\mathbf{\Psi}^{\mathrm{H}}\mathbf{\Psi} + \sigma^{\mathbf{2}}_{\mathbf{N}}\mathbf{I}\}^{-\mathbf{1}}\mathbf{\Psi}^{\mathrm{H}} \ , \tag{4}$$

where $\mathbf{\Psi}^{\mathrm{H}}\mathbf{\Psi}$ is a strongly diagonal matrix. This may result in a noise enhancement in case of ZF with full-beamforing due to near-zero values (weaker eigenbeams) along the diagonal of $\mathbf{\Psi}^{\mathrm{H}}\mathbf{\Psi}$. This problem can be mitigated using ZF together with a reduced beamforming scheme, *as will be discussed later*. MMSE also solves this problem for the case of full eigen-beamforming in the low to moderate SNR range.

III. UEP LOADING AND BEAMFORMING

A. Adaptive Loading Model

we consider a MIMO-OFDM system with N_T transmit antennas, N_R receiver antennas, and N subchannels. The two-dimension scheme in frequency and space is realized by a simple SVD at the receiver side, and is fed back to the transmitter. Without loss of generality, the diagonal in $\overline{\mathbf{D}}_k$ can be sorted in ascending order. Accordingly, the columns of the same beamforming matrix $\overline{\mathbf{V}}$ are sorted as well. This yields to different channel layers with different eigenbeams and different subcarrier qualities. These motivate for devoting N_g different data classes with different priorities onto sorted eigenbeams and subchannels. In principle, there are two extreme sorting alternatives (as in [1]):

- I- The intuitive method: use the subchannels with the highest SNR for the important data.
- II- The robust method: use the subchannels with the lowest SNR for the important data.

In order to proceed with either method I or II, all subchannels of all eigenbeams have to be combined and sorted together in ascending order. This makes the loading algorithm equivalent to the SISO model in [1]. The algorithm adaptively allocates subcarriers k, with power $P_{k,j}$ and bits $b_{k,j}$ to each class $j \in [1, N_g]$ allowing $\Delta\gamma_j$ dB margin separations.
For more details on this algorithm, the reader is referred to [2] and [1].

B. Power Loading

The power is allocated according to the modified water filling (MWF) in [7] using the notation in [1] as follows,

$$P_{k,j} = \min\left[\left(w - \frac{\sigma_n^2}{\lambda_{k,j}}\right)^+, \frac{\sigma_n^2(2^{b_{k,j}} - 1)}{\lambda_{k,j}/\gamma_j}\right], \quad (5)$$

where $(a)^+ \equiv \max(a, 0)$, w is the waterline, $\gamma_{k,j}$ are the noise margin for the k^{th} subcarrier and the j^{th} class, and $\lambda_{k,j}$ are the eigenvalues of $\mathbf{H}^{\text{H}}\mathbf{H}$. The first term in (5) represents the normal water filling, while the second term represents the effective power allocation needed for allocating $b_{k,j}$ bits with a noise level σ_n^2, eigenvalues $\lambda_{k,j}$, and noise margins γ_j.

C. Eigen Beamforming

Beamforming is implemented at the transmitter by multiplying the antenna array by a precoding matrix. This matrix maximizes the received power by directing the transmitted power to the optimum eigenbeams. The optimum precoding matrix is found to be the unitary matrix $\hat{\mathbf{V}}$ of the channel mean [5]. Moreover, weaker eigenbeams are suppressed to avoid bad performance. This can realized by switching some columns of this matrix $\overline{\mathbf{V}}$ to zero. Assuming the rank of the channel to be M, the order of n-D beamforming will be $0 \le n \le M$. Let us define

$$\overline{\mathbf{V}} = [\overline{\mathbf{V}}_1 \overline{\mathbf{V}}_2],$$

where $\overline{\mathbf{V}}_1 = [v_1, ..., v_n]$ and $\overline{\mathbf{V}}_2 = [0, .., 0]$.

Therefore, when $n = M$, the system realizes a full-beamforming, which is not necessarily the best case when the channel is highly correlated, i.e., has some weaker eigenbeams.

IV. RESULTS AND ANALYSIS

For generating these results, we assume an instantaneous channel with an error Ξ compared to the channel mean. The variance of this error $\sigma_\Xi^2 \in \{0, 0.1, 0.25\}$, where $\sigma_\Xi^2 = 0$ indicates the perfect CSI case. The simulated channel model considers two correlation schemes, the perfectly scattered ($\mathbf{R}_{tx} = \mathbf{R}_{rx} \approx \mathbf{I}$) channel that has the mean eigenbeam powers of $E\left[\mathbf{D}_v^2\right] = [10.5, 3.99, 1.29, 0.20]$, and the highly correlated (indoor-to-outdoor) channel that has the mean eigenbeams power of, $E\left[\mathbf{D}_v^2\right] = [15.81, 2.81 \times 10^{-1}, 1.7 \times 10^{-2}, 6 \times 10^{-4}]$ as in [9], where v is the spatial index $\in [0, 3]$.

For our simulations, we proposed a UEP profile that consists of 3 different classes of services, i.e., $N_g = 3$. The noise margin separations between each two classes Δ_j fixed to 3 dB, such that

$$\gamma_j = \gamma_0 + j\Delta_j \ , \quad (6)$$

where γ_0 stands for the highest priority class. The total number of bits for each class was chose to be $T_j = 1024$ bits, resulting in a total target bit rate of 3072 bits.

A 4×4 MIMO-OFDM system is considered, with an equivalent SISO subcarrier length of 512 subcarriers, i.e., each antenna accommodates for 512 subcarriers. The equivalent power for each antenna = $P_T/4$, where P_T is maximum permissible power emission from the transmitter.

A. Perfectly Scattered Channel

From the depicted results in Fig. 1, we can show that the full-length beamforming is enhancing the noise in the case of ZF equalization with CSI errors. The results are considerably improved by supressing the weakest eigenbeams, i.e., use shorter

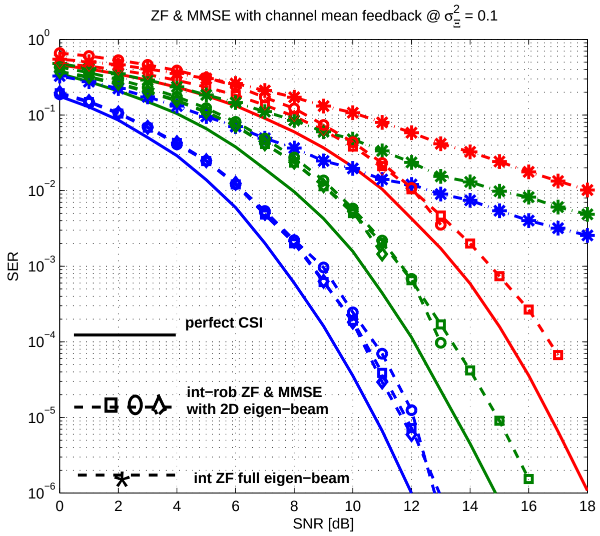

Fig. 1. Perfectly scattered channel with different beamforming equalization techniques

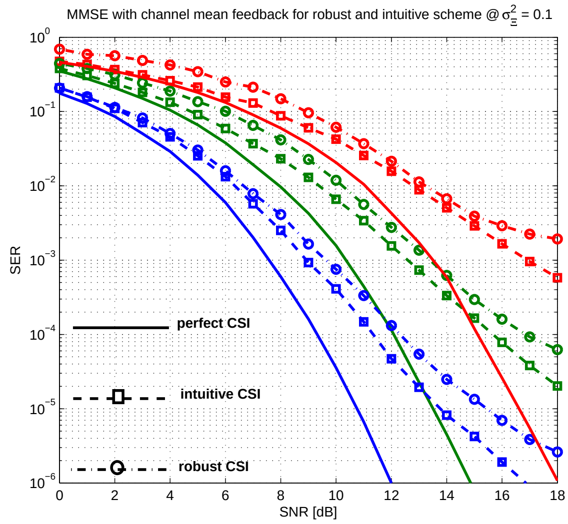

Fig. 2. Perfectly scattered channel using different sorting schemes and MMSE at a CSI error of $\sigma_\Xi^2 = 0.1$

beamforming (2-D beamforming). Exactly the same performance is observed for MMSE with a 2-D beamforming using either the intuitive or the robust scheme. This means that for CSI errors with $\sigma_\Xi^2 = 0.1$, a UEP system can realized with 3 dB margin separations with a loss of 0.8 dB.

In Fig. 2 at a $\sigma_\Xi^2 = 0.1$, the MMSE full-beamforming outperforms the ZF-full-beamforming (in Fig.1), assuming the previous channel conditions. However, an error-floor can be noticed at high SNR values. From the same figure, one can observe a 0.9 dB loss for robust bit loading (due to the statistical CSI errors), which is even less than the designed performance loss for the perfect case in [1]. In spite of this limited loss, the robust scheme is protected more against unexpected nonstationary noise (impulse noise) or sudden interferences, which could be an advantage over the intuitive loading. This would mean to guarantee, at least, the reception of the high priority data under these tough conditions.

Using a higher CSI error, $\sigma_\Xi^2 = 0.25$, the full eigen-beamforming introduces a loss of (**8 dB** at 10^{-4} symbol error rate) compared to the perfect CSI case (see Fig. 3). This deterioration could not be corrected using the error compensation method in [7] with a fixed power regime and a rate reduction by 10%, i.e., as a first guess. The compensation is done by reducing each eigenvalues, used in the power allocation, by $N_T\sigma_\Xi^2$.

By switching to the 2-D beamforming, the performance gab is mitigated. In Fig. 4, the 2-D beamforming reduces the previous gab to almost 3 dB

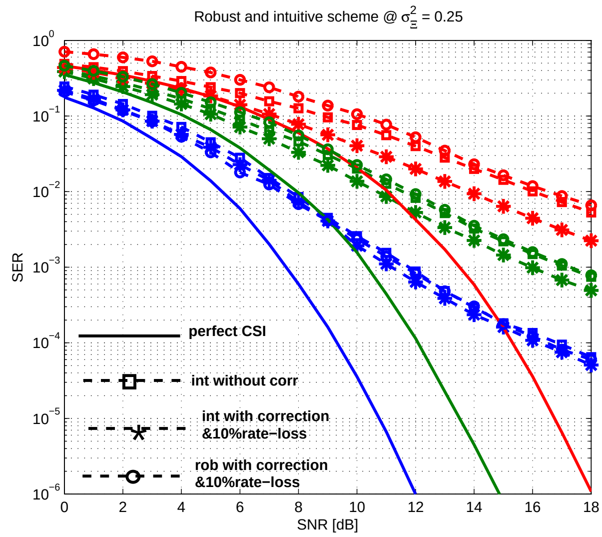

Fig. 3. Perfectly scattered channel with different sorting schemes at CSI error $\sigma_\Xi^2 = 0.25$ and error compensation

in case of intuitive bit-loading, although there is an undesired error-floor at higher SNRs. The robust scheme is performing slightly worse (0.4 dB) than the intuitive scheme in this case. Nevertheless, the SER curves show no error-floor at high SNRs and full compliance with the proposed UEP profile.

B. Highly Correlated Channel

The performance of the highly correlated channels is too scarce for realizing an efficient MIMO-OFDM adaptation. Although these channels are not interesting for adaptive MIMO, one should design the adaptation scheme such that it could conserve the

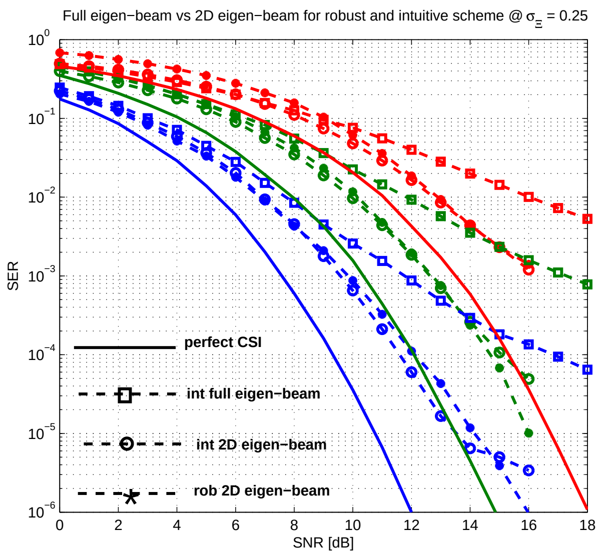

Fig. 4. Perfectly scattered channel with different beamforming and sorting schemes at CSI error of $\sigma_{\Xi}^2 = 0.25$

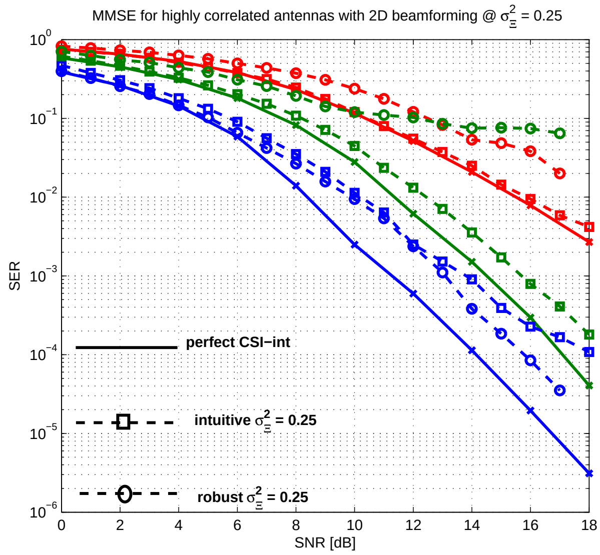

Fig. 5. Highly correlated channel with 2D beamforming at CSI errors of $\sigma_{\Xi}^2 = 0.25$

performance of the mandatory information when the system suddenly deviates to such a case. However, the less scattered channels are practically reduced to a single beamforming (1-D) scheme, i.e., the water-filling will dynamically allocate zeroes to the weaker subcarriers.

As seen in Fig. 5. The robust scheme performs, for the highest priority class, close to the perfect case. Nevertheless, it sacrifices the lower protected classes, which could be compensated by reducing the data rates. The intuitive scheme shows an error floor at the highest priority class, although it preserves the performance of the other classes.

V. Summary

We described a UEP bit-allocation scheme for MIMO-OFDM as a two-dimensional channel adaptation by allocating arbitrary number of bits with different priorities to eigenbeams and frequencies. This algorithm allows also arbitrary margin definitions according to a given UEP profile.

The decreased order beamforming enhances the performance compared to the full length beamforming in case of CSI errors and high antenna correlation. This means that the MIMO channel size is practically reduced.

Finally, the proposed spatial UEP bit-loading algorithm ensures higher performance to the higher priority data, especially, when a robust loading is used. This protects the system against sudden CSI variations or non-stationary noise.

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