Synchronization in Multicarrier Systems
[an overview]

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Carrier Recovery involves
  - Estimation of carrier synchronization parameters.
  - Correcting the received carrier signal.

Synchronization parameters:
  Carrier frequency offset and Carrier phase offset

**Carrier frequency offset:**
  is due to Frequency instability in transmitter or receiver and the Doppler effect

![Graph](image-url)  
16 QAM with Frequency Offset
Carrier Recovery 2

Carrier Phase offset:
Phase instability in clock signal generator, Phase due to transmission delay and Thermal noise [AWGN]

16 QAM with Phase Offset
Carrier recovery involves two stages of operation to reduce the amount of computational effort.

- **Carrier Acquisition**
- **Steady state tracking**

**Carrier acquisition:**
- Transition from a large initial uncertainty about the synchronization parameters to a small steady state estimation error variance.
- It involves Carrier frequency offset and phase offset estimation and correction.
- Carrier acquisition is a coarse estimation of the value

**Carrier Tracking:**
- Carrier tracking involves the fine tuning of the coarse estimate of the offsets obtained in acquisition.
Effects of Phase and Frequency Offset

- Frequency offset results in the phase rotation of the carrier signal.
- Frequency offset is equivalent to the phase derivative: \( \delta \omega = \delta \psi / \delta t \).
- When the sampling rate is wrong in the case of DMT, this would also mean a frequency offset at every carrier which increases with the frequency. Thus, phase rotation will increase with frequency.

![Perfect sampling vs Frequency Error](image)

- In case of sampling in the time domain the error increases proportionally with the carrier frequency [symbol period reduces inversely with the carrier frequency].
• The prefix should be greater than channel delay length to avoid intersymbol interference.
• The channel delay introduces a phase offset proportional to the time delay experienced.
• At the receiver the guardband is stripped off and the signal is detected after delay and frequency offset compensation.
Time domain shift \( \Leftrightarrow \) frequency domain rotation

- The effect of sample phase shift results in a rotation of FFT outputs.
- The sample phase shift can be compensated by rotating each FFT output over an angle proportional to the estimated phase shift ‘\( \varepsilon \)’.

Effect of a sample phase shift by half a sampling period
• The baseband samples are obtained by oversampling of a known preamble.
• The samples are used to acquire the initial frequency offset estimate and phase offset estimate sequentially.
• Frequency offset is estimated and signal is compensated before entering the phase offset estimator.
• The frequency offset can be compensated by multiplying the baseband samples by complex number $e^{-j2\pi \delta f \cdot kT_s}$ in the phase rotator.
• The phase offset is corrected by multiplying the frequency synchronizer output by $e^{-j\phi'}$ after the phase offset $\phi'$ is estimated.
Synchronization in DSL

• Open Loop Timing Recovery
• Pilot Insertion Timing Recovery
• Decision - Directed Timing Recovery
• Pointer Add/Delete Mechanisms
• Extraction of symbol rate frequency and phase from the channel output at the customer premise modem.

• Initial synchronization is achieved by the transmission of mid-band sinusoid from the central office to the remote modem

\[
\sigma_e^2 << \frac{1}{4\pi^2 f_m^2 \cdot \text{SNR}}
\]

\(f_m\) - signals with frequency equal to less than \(f_m\)
\(\sigma_e\) - time error standard deviation
Initial synchronization is achieved by the transmission of mid-band sinusoid from the central office to the remote modem.

Remote modem samples at what it expects to be the zero crossings and the signal level at the sampling instants forms the phase error to be used by PLL for correction.

**Loop timing:**
- clock recovered by the remote modem is reused for the upstream transmission
- used for both data detection and modulation in the reverse direction
- non-PLL end modem use its transmit sample clock to derive its receive sample clock
Timing recovery is known as **square-law** timing

- Its noise performance is near-optimum, especially at low SNR.
- A sinusoidal input results in a sinusoidal output at double the frequency.
- A square law is mathematically easily tractable.

- It requires at least three samples per symbol.
- Works fine with time-limited waveforms but self noise arises with bandlimited waveforms.

- Self noise can be removed by introducing suitable pre and post filters.
Pilot Insertion Timing Recovery

- **Pilots** are inserted at known frequencies into the transmitted waveform $x(t)$.

- **Pilot training** includes repeated known patterns in the transmitted waveforms and carries sufficient information for the receiver to determine the phase error with respect to the local estimate of the symbol rate.

- The pilot frequencies are filtered in the receiver and passed to PLL to generate sampling and symbol rate clocks.

- Phase error is easily computed as filtered sample values at the estimated zero crossing sample times of the pilot.

- The drawback is loss of bandwidth by the introduction of pilot signals into the useful transmission band.
Decision - Directed Timing Recovery

- Channel outputs and previous/current symbol decision are used to generate the phase error.
- Does not generate a local reference sinusoid (pilots) – saving of bandwidth.
- Employs MMSE criteria over the sampling phase for estimation.
Autonomous Synchronization of DMT-VDSL System

- Synchronization for DMT-Zipper based VDSL modems in an unbundled networks

**Zipper Duplex Method:**

- It uses different DMT carriers in different transmission directions.
- The allocation is done dynamically enabling run-time adaptation of bit rates.
- Traditional DMT uses *cyclic prefix* to preserve orthogonality between subcarriers,
  Zipper adds extra *cyclic suffix* to preserve ortho’ty between upstream and downstream.
- Two network ends should be synchronized in time and frequency to maintain orthogonality.
- Sampling Frequency synchronization between network ends to ensure subcarrier spacing.
- With this scheme, a terminal receives intended signal and also NEXT from nearby Txm’s.

\[
r_{k}(t) = [x_{up,k}(t)*h_{up,k}(t)] + [x_{up,k}(t)*h_{NE,k}(t)] + v_{NEXT,k}(t) + n_{k}(t)
\]

- Desired signal
- Near Echo
- NEXT from other modems
- AWGN
Correlation signal \( \rho_k(\theta) \), for a sum of three DMT signals

\[
\theta_{av,k} = \frac{1}{(k-1)} \sum_{\ell = 1, \ell \neq k} (\theta_\ell - \theta_k)
\]

\[
\hat{\theta}_{ML} = \arg \max_{\theta} \{ \Lambda(\theta) \}
\]

\[
\hat{\theta}_{ML} = \arg \max_{\theta} \left\{ \sum_{m=0}^{M-1} \Lambda_m(\theta) \right\}
\]
ADSL and VDSL
If the bit timing base of user data streams is not synchronous with ADSL modem timing base, the input data streams shall be synchronized to the ADSL timing base using the synchronization control mechanism.
• Synchronization control for the fast data buffer may occur in frames 2 through 33, and 36 through 67 of an ADSL superframe.

• The fast byte may be used as the synchronization control byte.
• Synchronization control for the interleaved data buffer can occur in frames 1 through 67 of an ADSL superframe.

• No synchronization action is taken during frame 0, where the sync byte is used for CRC.
Pilot tone
- During initialization VTU-R select a sub-channel for timing recovery.
- VTU-O transmits the 4QAM value of 00 on pilot tone during every symbol.

Timing Advance
- It is equal to the transmission delay from the VTU-O to the VTU-R.
- The TA forces the VTU-O/VTU-R pair to start transmissions of frames in opposite directions simultaneously.
- The TA is subtracted from the received symbol start time, and the result is used as the VTU-R’s individual symbol start time so that both the VTU-O and VTU-R transmitters start transmitting each DMT frame at the same time.

Synchronous Mode
- In synchronous mode, all VTU-O transceivers on the same cable binder shall transmit with respect to a common symbol clock, and thus start the transmission of DMT symbols at the same time.
- The symbol clock is derived from a reference clock.
**Loop timing**

The VTU-R shall loop time its local sampling clock to the pilot during initialization.

*fig: Physical Medium Dependant [PMD] frame format*
Synchronization in OFDM

- Time Domain Synchronization
- Frequency Domain Synchronization
1. Accurate demodulation and detection of an OFDM signal requires sub-carrier orthogonality.

2. Variations of the carrier oscillator frequency, the sample clock, or the symbol clock affects the orthogonality of the system.

3. Symbol time offset and frequency offset may cause intersymbol and intercarrier interference.

4. The drift in the sample clocks of the transmitter and the receivers produce negligible degradation effects to the system performance.

5. The main focus is on the study of “frequency offset and time offset” estimation and recovery.
Carrier acquisition Phase:

- Initial estimate of the errors is acquired using more complex algorithms and with a higher amount of synchronization information in the data signal.
- Carrier acquisition is a coarse estimation of the value.

Carrier Tracking Phase:

- Involved corrections of short-term deviations.
- Carrier tracking involves the fine tuning of the coarse estimate of the offsets obtained in the carrier acquisition phase.
Time domain Synchronization
1. To estimate and correct symbol and frame offset in time domain
2. To reduce inter OFDM symbol interference [ISI] and Phase error between subcarriers due to FFT window misalignment.

Frequency domain Synchronization
1. To estimate and correct frequency offset in subcarriers
2. To reduce intercarrier interference [ICI]

OFDM Synchronization Steps

- **Time domain**
  - **Symbol synchronisation**
  - **Fractional frequency offset**
  - **Correlation of periodic signal**

- **Frequency domain**
  - **Integer frequency offset**
  - **Fine time offset**
  - **Energy location of impulse response**
  - **Frequency offset tracking**
  - **Decision-directed or pilot-based control loop**
The phase error evolved out of the time offset is mainly depending on the modulation scheme employed.

\[
\begin{align*}
    f(t) & \leftrightarrow F(\omega) \\
    f(t-t_0) & \leftrightarrow F(\omega) e^{j\omega t_0}
\end{align*}
\]

1. Coherent Modulation
- suffers from the FFT window misalignments
- results in loss of reference phase which makes coherence detection erroneous
- needs phase correction mechanisms

2. Pilot Symbol assisted Modulation
- pilots are interspersed with the data symbols in the frequency domain
- receiver can estimate the evolving phase error from the pilots phases
- the number of pilots depends on the maximum anticipated time shift \( \tau \)
- distance between two pilot tones in OFDM spectrum is found to be
  \[
  \Delta P \leq N/2m
  \]
- wideband channel estimation technique is employed
- reduces spectral attenuation and the phase rotation

3. Differential Modulation

- implemented either between corresponding subcarriers of consecutive OFDM symbols or between adjacent subcarriers of the same OFDM symbol.

Time Domain Synchronization Errors - Summary

- Misalignment of the receiver’s FFT window relative to the sample stream leads to inter OFDM symbol interference.
- It also results in the shift of the reference phase throughout the received frequency domain OFDM symbol.
- The effects of the inter OFDM symbol interference can be reduced by introducing cyclic preamble to the OFDM symbol.
- The performance degradation from the phase errors can be improved by using differential modulation.
Synchronization Algorithms

- Down sampling and the clock recovery module DS determines the optimum sampling instant.
- Time synchronization TS controls the frequency acquisition FA, Frequency tracking FT as well as the timedomain alignment of the FFT.
Synchronization Algorithms

1. Coarse frame and OFDM symbol synchronization
2. Fine symbol tracking
3. Frequency acquisition
4. Frequency tracking
5. Time and frequency-domain synchronization based on Autocorrelation
6. Multiple access frame structure
7. Frequency tracking and OFDM symbol synchronization
8. Frequency synchronization and Frame synchronization

Courtesy:
Single and Multicarrier Quadrature Amplitude Modulation, by L.Hanzo, W.Webb, T.Keller
Pan European DVB system:

• uses a so-called null symbol as the first OFDM symbol in the time frame.
• no energy is transmitted during the null symbol and it is detected by monitoring the received baseband power in the time domain.

An other method uses

• bursts of at least three OFDM symbols per time frame.
• Two of the symbols in the bursts contain synchronization subcarriers bearing known symbols.
Fine symbol tracking [2]

generally based on the correlation operation either in time or frequency domain

- *Warner and Bingham* employed frequency domain correlation of the received synchronization pilot tones with known synchronization sequences.

- *Couasnon* utilized the redundancy of the cyclic prefix by integrating over the magnitude of the difference between the data and the cyclic extension.

- *Sandell* proposed exploiting of the received time domain samples of the cyclic extension for time domain tracking.
Frequency acquisition

It provides an initial frequency error estimate to be used for tracking

- *Sari* proposed use of pilot tones embedded into the data symbol surrounded by zero valued virtual carriers, so that frequency shift can be easily located.

- *Moose* suggested a repeated OFDM symbol pair and use of shorter DFT for increasing the subcarrier distance and thus the frequency error estimation range is extended.

- *Clazen* proposed the use of pseudo noise carried by synchronization subcarriers and the frequency acquisition is performed by searching the sequence.
Frequency tracking \[4\]

relies on the already established *coarse frequency estimation*

- *Moose* proposed the use of phase difference between the subcarriers of repeated OFDM symbols to estimate the frequency deviations.

- *Clazen* employed frequency domain synchronization subcarriers embedded into the data symbols for which phase shift between the consecutive OFDM symbols are measured.

- *Daffara* and *Sandell* used the phase of the received signal’s autocorrelation function, which represents the phase shift between received data samples and their repeated copies in the cyclic extension of the OFDM symbols.
- Carrier frequency offset $\delta f$ results in phase error $\psi$ of the received OFDM sample

$$
\psi(\delta f) = 2\pi \cdot \delta f \cdot T_s
$$

$$
= (2\pi \cdot \delta f) / (N \cdot \Delta f)
$$

$N$ - no of subcarriers

$\Delta f$ - subcarrier spacing

- Phase error difference between two time domain samples is a function of frequency error and their time delay.

$$
\psi_1(\delta f) - \psi_2(\delta f) = \psi_{1-2}(\delta f)
$$

Phase error can be used to determine frequency error.

- Phase at $G_{\text{max}}$ equals the phase shift between the guard band samples and corresponding data samples of current OFDM symbol.

$$
\delta f = (\Delta f / 2\pi) \cdot \angle G_{\text{max}}
$$

$G_{\text{max}}$ – correlation between two sequences of $N_g$ samples length.
Exploit the correlation introduced by the guard interval:

\[ r_{m+n} = r_m \cdot e^{j2\pi f} \]

OFDM symbol

\[ \text{OFDM symbol} \]

Symbol time Estimation
Maximum Likelihood Estimation

Random sample - $X_1, X_2, \ldots, X_n$
Probability function - $p(X_k, \theta)$
Probability density function - $f(X_k, \theta)$
Unknown parameter(s) - $\theta$
MLE is a particular technique for estimating the value(s) of the $\theta$

Likelihood Function

Joint probability (density)

$$L(X, \theta) = \prod_{k=1}^{n} p(X_k, \theta)$$

$$L(X, \theta) = \prod_{k=1}^{n} f(X_k, \theta)$$
Maximum Likelihood Estimate

The Maximum Likelihood Estimate is the value(s) of the parameter(s) $\theta$ which maximize the likelihood function

$$L(X, \theta) = \prod_{k=1}^{n} f(X_k, \theta)$$

Log-likelihood

Logarithm function is monotone, so MLEs also maximize the log-likelihood function

$$\log L(X, \theta) = \log \prod_{k=1}^{n} f(X_k, \theta) = \sum_{k=1}^{n} \log f(X_k, \theta)$$
Maximum Likelihood Estimation

Score

The derivative of the log-likelihood function is called the **score**

\[ s(X, \theta) = \frac{d}{d \theta} \log L(X, \theta) \]

Observed Information

The second derivative of the **negative** log-likelihood function is called the **observed information**.

\[ I(X, \theta) = -\frac{d^2}{d \theta^2} \log L(X, \theta) \]

MLE maximize the log-likelihood function so

1. The **score** (derivative) must be **zero**
2. The second derivative must be **negative**
Maximum Likelihood Estimation

When the random sample is Normally distributed

\[ f(x_k, \theta) = \frac{1}{\sigma_k \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x_k - \mu_k(\theta)}{\sigma_k} \right)^2 \right] \]

\[ \log L(X, \theta) = \sum_{k=1}^{n} \log \left( \frac{1}{\sigma_k \sqrt{2\pi}} \right) - \frac{1}{2} \sum_{k=1}^{n} \left( \frac{X_k - \mu_k(\theta)}{\sigma_k} \right)^2 \]
Minimum Mean Square Estimation

**Estimation**: Estimating the value of an inaccessible random variable $Y$ in terms of the observation of an accessible random variable $X$.

$$g(X) : \text{the estimate for } Y \text{ is given by a function of } X.$$  

Estimation error: $Y - g(X)$  

Cost function: $C(Y - g(X))$

We are interested in finding the function *minimizes* the expected value of the cost.

$$\min_{g(X)} \left( E \left[ C(Y - g(X)) \right] \right)$$
Minimum Mean Square Estimation

Example: Estimate $Y$ by $g(X) = aX + b$, by means of MMSE criterion:

$$\min_{g(X)} \left( E \left[ (Y - g(X))^2 \right] \right) = \min_{a,b} \left( E \left[ (Y - aX - b)^2 \right] \right)$$

$$b^* = E[Y - aX] = E[Y] - aE[X]$$

$$\Rightarrow \min_{a,b} \left( E \left[ (Y - aX - b)^2 \right] \right) = \min_a \left( E \left[ (Y - E(Y)) - a(X - E[X]) \right] \right)$$

$$0 = \frac{d}{da} E \left\{ (Y - E(Y))^2 - 2a(Y - E(Y))(X - E[X]) + a^2 (X - E[X])^2 \right\}$$

$$0 = -2E \left\{ (Y - E(Y))(X - E[X]) \right\} + 2aE \left\{ (X - E[X])^2 \right\}$$

$$2aE \left\{ (X - E[X])^2 \right\} = 2E \left\{ (Y - E(Y))(X - E[X]) \right\}$$
\[ a^* = \frac{E \left\{ (Y - E(Y)) (X - E(X)) \right\}}{E \left\{ (X - E(X))^2 \right\}} = \frac{COV(X,Y)}{VAR(X)} = \rho_{xy} \frac{\sigma_y}{\sigma_x} \]

\[ \rho_{xy} = \frac{COV(X,Y)}{\sigma_x \sigma_y}, \sigma_x = \sqrt{VAR(X)}, \sigma_y = \sqrt{VAR(Y)} \]

The minimum mean square error (MMSE) linear estimator:

\[ \hat{Y} = a^* X + E \left[ Y - a^* X \right] = a^* [X - E(X)] + E[Y] \]

\[ = \rho_{xy} \sigma_Y \left( \frac{X - E[X]}{\sigma_X} \right) + E[Y] \]

If \( X \) and \( Y \) are uncorrelated, \( \rho_{xy} = 0 \) \( \Rightarrow \hat{Y} = E[Y] \)
Thank you