Unequal Error Protection with Eigen Beamforming for Partial Channel Information MIMO-OFDM

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Abstract— **Some communication applications, like multimedia, deliver data of different importance classes allowing unequal error protection (UEP) levels. In this paper, a multiple-input multiple-output (MIMO) system using orthogonal frequency division multiplexing (OFDM) is considered with a new UEP bit-loading algorithm based on the non-UEP algorithm by Chow, Cioffi, and Bingham. In the proposed bit-loading algorithm, bits are distributed across the unitarily transformed eigenbeams in case of partial channel information (CSI) knowledge. However, ideal performance can not withstand the rapid wireless channel variation unless a restricted beamforming is used. For the case of partial CSI, we proposed a switched beamforming technique that can maintain the required performance protection levels.**

I. Introduction

The source encoders of some communication applications, like multimedia, deliver data of different importance levels, i.e., different data requires different levels of protections. Such applications are demanding for UEP, in which important data is protected more against errors. In turn, MIMO channel can easily realize different eigen channel-beams with different qualities, which can be used as a backbone for the variation of protection levels. Therefore, it is necessary to design new techniques that adapt the resources at the modulation scheme, the code rate, and the spatial diversity such that the overall performance satisfies a certain UEP profile.

Due to its suitability for adapting individual carriers and individual eigen channel layers with different bit-rates, MIMO-OFDM is selected in this work to realize different levels of protections. In MIMO-OFDM, the available bandwidth is divided into N subchannels. The number of bits per subchannel is adaptively tuned to fit the different eigen-channel qualities, such that subchannels located in the strongest eigen channel are loaded with a high number of bits. This process is called *bit-loading*.

In the single-input single-output (SISO) case a number of algorithms has been developed to solve the bit-loading problem in both wireline and wireless environments. Hughes and Hartogs [2] and Campello [3] have proposed discrete bit-loading algorithms that successively allocate bits to subchannels that require the minimum incremental power. To reduce the complexity of this discrete algorithms, Chow et al. [4] and Fischer et al. [5] have proposed their sub-optimal algorithms that load the subchannels according to their signal-to-noise ratio (SNR) and the required noise margin γ_m based on Shannon capacity or based on Lagrange multipliers for minimizing the symbol-error probability.

Based on these previous algorithms, many approaches for achieving adaptive MIMO-OFDM in case of perfect CSI are developed in [7], such that bits are allocated across the different subchannels on each eigen channel. However, it is more realistic to consider a partial CSI, in which the amount of feedback is reduced by either transmitting part of the channel information [9] or the channel statistics over a certain period [8], [10], [11]. In both cases, the partial CSI at the transmitter is considered to be the deterministic information for bit-loading at the transmitter. This results in an instantaneous channel decomposition error between the current effective channel and the deterministic one, due to the rapid wireless channel variations or the errornous feedback channel.

The previous algorithms [7-11] achieve a constant error probability for every eigen-channel, which is not required for many applications like video and audio. Since the physical transport already allows for an easy implementation of UEP properties, we propose an UEP bit-loading algorithm that allocates bits according to the eigen-channel gain and the required protection level based on [1]. In this algorithm, we satisfy arbitrary performance margins between the protection classes' as well.

In contrast to [1], the most important data is allocated to subchannels with the highest eigen-channel gain, as the induced interference due to the channel feedback errors [13], [12] has a Gaussian distribution, which differs from the impulse noise case impulse noise in [1]. Finally, a switched beamforming scheme is proposed to reduce the induced interference due to channel error, i.e., we select the eigenvectors to avoid transmission on the highest interfered eigen-beam.
The rest of the paper is organized as follows: Section 2 describes the adaptation principles in MIMO-OFDM with partial CSI. Section 3 states the proposed bit-loading algorithm. Section 4 discusses some simulation results. Finally, the paper in concluded with Section 5.

II. UEP Adaptive MIMO-OFDM

Multiple-input multiple-output (MIMO) techniques can ideally provide independent channels just like OFDM when SVD is used for diagonalization. As shown in [1], the UEP bit and power loading for single-input single-output (SISO) OFDM

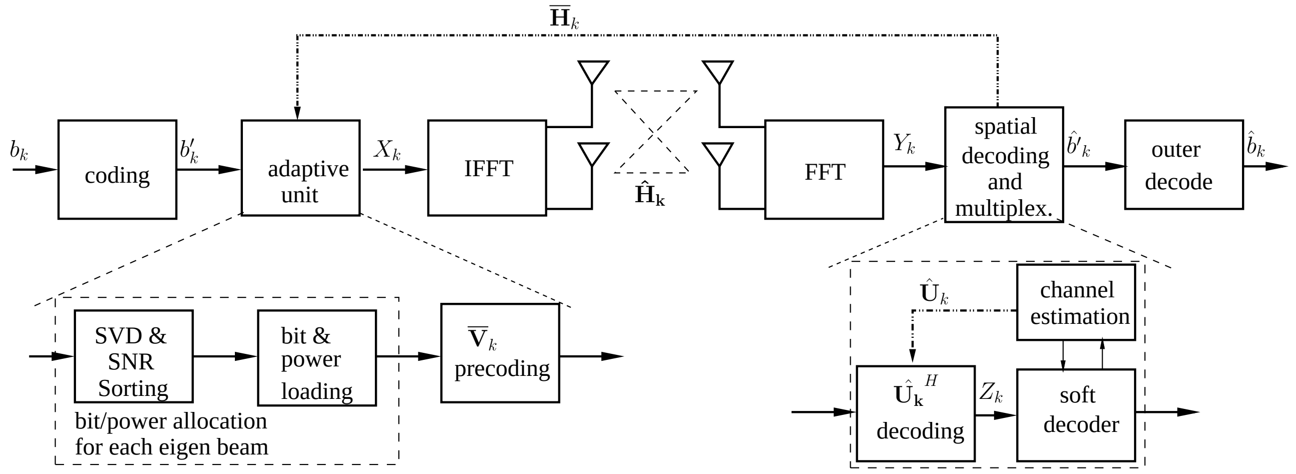

Fig. 1. MIMO UEP bit-loading

system are readily available. However, for MIMO-OFDM system it will be more complicated, as the adaptation to the channel matrix $\mathbf{H}$ is done by resolving the SNR seen by M transmitter and R receiver antennas in two dimenssion (frequency and space). The singular value decomposition (SVD) is used here as a tool to separate the MIMO channels $\mathbf{H}$ into parallel independent SISO channels. Later, the total SNR for every subchannel in each eigenbeam is being used to load bits and power according to the required UEP profile.

A. Channel Model

In our system, we assume a MIMO channel matrix $\mathbf{H}_k$ for each subchannel k. This matrix which is given by a zero-mean complex Gaussian distributed $M \times R$ matrix, assuming a flat fading response with channel variations faster than the feedback rate. This channel has a statistical mean of $\overline{\mathbf{H}}_k$. The antennas of the transmitter and the receiver are correlated by the correlation matrices $\mathbf{R}_{\text{tx}}$ (transmitter antennas correlation) and $\mathbf{R}_{\text{rx}}$ (receiver antennas correlation). The equivalent channel $\mathbf{H}_e$ is given by

$$\mathbf{H}_{e,k} = \mathbf{R}_{\text{tx}}^{1/2} \cdot \mathbf{H}_k \cdot \mathbf{R}_{\text{rx}}^{1/2} \ . \tag{1}$$

B. Partial CSI

Many applications already use a periodic feedback between the receiver and the transmitter in order to keep the communication link updated with the current channel information. However, the channel information can not be completely provided to the transmitter, due to the limitation in the transmission capacity or the rapid channel variations. Therefore, it is more convenient to transmit a partial CSI as in [8], [9], and [10]. Hence, the mean feedback of the channel $\overline{H}$ is transmitted back to the transmitter, which acts as a partial CSI. The channel mean $\overline{\mathbf{H}}$ is assumed to be deterministic at the transmitter side, while a perfect CSI is assumed at the receiver side. Using these assumptions, we have optimized a UEP bit-loading algorithm in order to devote a number of protection classes N_g to the realized eigen channels.

C. System Model

As depicted in Fig. 1, we considered a MIMO-OFDM system with M transmit antennas (M-IFFTs), R receiver antennas (R-FFTs), and N subchannels. The encoded data are sorted according to the SNR of each subchannel across the eigenbeams, which are realized by SVD. To realize different UEP classes, the sorted SNR ranges have to be allocated to the required protection levels N_g. In principle, there are two sorting mechanisms (as in [1]):

I- The intuitive method: use the subchannels with the highest SNR for the important data.

II- The robust method: use the subchannels with the lowest SNR for the important data.

In order to procede with either method I or II, all subchannels, of all eigenbeams, have to be combined in a long buffer (assuming a long SISO case). The sorting algorithms have to go through this buffer sequentially in order to satisfy the two dimenssion sorting (see Figure 2). Furthermore, hypothetical SNR thresholds τ_j are set within this buffer such that the UEP requirements are fulfilled by modifying these thresholds, thereby changing the number of subchannels of each class.
Hence, subchannels in each protection class set M_j are allocated according to a given UEP profile and a feedback channel mean $\overline{\mathbf{H}}_k$. The allocated beams are transmitted through the beamforming precoding matrix $\overline{\mathbf{V}}_k$, which is the right hand side unitray matrix of the SVD of $\overline{\mathbf{H}}_k$, where the SVD of $\overline{\mathbf{H}}_k$ is given by $\overline{\mathbf{H}}_k = \overline{\mathbf{U}}_\mathbf{k}\overline{\mathbf{D}}_\mathbf{k}\overline{\mathbf{V}}_\mathbf{k}^{\ H}$. $\overline{\mathbf{V}}_k$ and $\overline{\mathbf{U}}_k$ are unitary matrix of size $M \times M$ and $R \times R$, respectively. $\overline{\mathbf{D}}_k$ is a diagonal matrix consists of singular values of $\mathbf{H}_k$ arranged in ascending order. At the same time, $\overline{\mathbf{V}}_k$ indicates the eigen vectors for the k^{th} subchannel of $\overline{\mathbf{H}}_k\overline{\mathbf{H}}_k^H$. The rank of $\overline{\mathbf{H}}$ is given by $\min(R, M)$, and for simplecity, we assume here that $R = M$.
The received vector $\mathbf{Y}_k$ for the k^{th} subchannel can be written as

$$\mathbf{Y}_k = \hat{\mathbf{U}}_k^H \hat{\mathbf{H}}_k \overline{\mathbf{V}}_k \mathbf{X}_k + \overbrace{\hat{\mathbf{U}}_k^H \cdot n_k}^{\text{noise term}} \ , \tag{2}$$

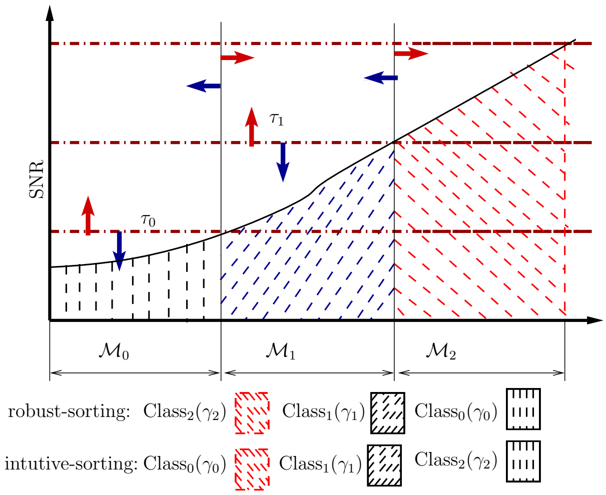

Fig. 2. SNR thresholds for three protection classes assuming Class$_0$ is the highest protected class in both schemes.

where $\mathbf{X}_k$ is the transmitted vector and $\hat{\mathbf{H}}_k$ is the instantaneous frequency domain channel matrix, which is adopted from [10] and [12] to be

$$\hat{\mathbf{H}}_k = \overline{\mathbf{H}}_k + \Xi_k \ , \tag{3}$$

where Ξ_k represents the CSI error, due to either feedback delay or link-error, which is assumed to be a zero-mean Gaussian with a variance σ^2_{Ξ}. The SVD of the instantaneous channel matrix on the k^{th} subchannel is written as $\hat{\mathbf{H}}_k = \hat{\mathbf{U}}_k\hat{\mathbf{D}}_k\hat{\mathbf{V}}_k^H$. Then the received vector is given by

$$\begin{aligned} \mathbf{Y}_k &= \overbrace{\hat{\mathbf{U}}_k^H\hat{\mathbf{U}}_k}^{\mathbf{I}}\hat{\mathbf{D}}_k\overbrace{\hat{\mathbf{V}}_k^H\overline{\mathbf{V}}_k}^{\mathbf{\Psi}}\mathbf{X}_k + \eta_k \\ &= \hat{\mathbf{D}}_k\mathbf{\Psi}\mathbf{X}_k + \eta_k \ , \end{aligned} \tag{4}$$

where $\mathbf{\Psi}_k$ represents the noisy identity matrix due to the insufficient CSI. $\mathbf{\Psi}_k$ is almost diagonal matrix, while a closed form expression for $\mathbf{\Psi}_k$ cannot be found [12], even for a smaller values of σ^2_{Ξ}. $\hat{\mathbf{D}}_k\mathbf{\Psi}_k$ represents the total power on each eigenbeam (on the diagonal) and the interfering power from the adjacent eigenbeams due to the CSI error. The interference patterns are related to the channel correlation matrix $\mathbf{R}_{\text{corr}}$, such that the interference of the strongest eigenbeam is critical when the channel is a highly correlated channel.

III. UEP: New Bit-Loading Principle

A. The New UEP Bit and Power Loading Algorithm

Assuming a bit-rate maximization problem (BRMP), our algorithm is based on the one by Chow, Cioffi, and Bingham [4], which uses an adaptive noise-margin to fulfill the target rate. We also use margins γ_j for the different priority classes. Hence, the algorithm uses the rate relation

$$b_{k,j} = \log_2\left(1 + \frac{\text{SNR}_{k,j}}{\gamma_j}\right)$$

$$\hat{b}_{k,j} = \begin{cases} \lfloor b_{k,j} + 0.5 \rfloor, & \text{if } b_{k,j} \le b_{max} \\ b_{max}, & \text{if } b_{k,j} > b_{max} \end{cases}$$

$$\Delta b_{k,j} = b_{k,j} - \hat{b}_{k,j} \ , \tag{5}$$

where $b_{k,j}$ is the bit rate for the k^{th} carrier in the j^{th} priority class, such that, e.g., $j \in [0, \ldots, N_g - 1]$ would correspond to N_g protection levels. γ_j is fixed for a set of subchannels $k \in \mathcal{M}_j$. The number of bits, $b_{k,j}$ are truncated to the nearest integer $\hat{b}_{k,j}$, where $\Delta b_{k,j}$ is the 'quantization error', and b_{max} are the maximum allowed bits per subchannel. Each group is composed of a certain number of bits, T_j, and the total target bit-rate is given by $B_T = \sum_{j=0}^{N_g-1} T_j$. We consider constant noise margin steps $\Delta\gamma_j$ between the groups. A generalization to different step sizes is obvious.

The complete algorithm is the following:

UEP Bit-Rate Maximization Algorithm
Input: SNR$_{k,j}$ in k^{th} subchannel of j^{th} class, N, N_g, B_T, T_j, and $\Delta\gamma$
Output: γ_j, average probability of error $\hat{\mathcal{P}}_{ej}$, and bit allocation

1) transmitter sorts the subchannels positions in case of "SNR-sorting".
2) ordered subchannel indices are stored in the set $\mathcal{M}_j$, where $j \in [0, \cdots, N_g - 1]$
3) γ_0 may initially be set to 1 and is iteratively adjusted to fulfill the UEP requirements and the required individual number of bits T_j. Let γ_0 denote the highest protection level. The others are computed as $\gamma_j = \gamma_0 - j \cdot \Delta\gamma$ in dB
4) $b_{k,j}$ is calculated using (5), then number of subchannels in $\mathcal{M}_j$, $j = 0, \ldots, N_g - 2$ is adjusted iteratively using a binary search as described in [1]. This process is equivalent to sliding the SNR thresholds, τ_j.
5) If the overall bit rate B_{tot} is not equal to the target bit rate B_T, γ_0 is recalculated using the following adjustment, as in [4], $\gamma_{0,new} = \gamma_{0,old} \cdot 2^{\frac{B_{tot}-B_T}{N}}$ and γ_j, $j > 0$ are again calculated as in Step 3). Then, go to Step 4).
6) Else, if B_T is fulfilled and/or the maximum iteration count is reached, go to Step 7)
7) If the maximum number of iterations is approached without achieving B_T, brute-force measures, as in Step 8) and Step 9) of [4], are taken. Dependent on $\Delta b_{k,N_g-1}$, bits are added to the least protected class at locations of maximum $\Delta b_{k,N_g-1}$ or bits are removed at locations of minimum $\Delta b_{k,N_g-1}$ until the target bit-rate is fulfilled.
8) The power is allocated using the symbol-error rate equations in [1], such that the power for each subchannel is a function of the noise power at each subchannel $N_{k,j}$, the rounded number of bits $\hat{b}_{k,j}$, and the required average error probability $\overline{\mathcal{P}}^e_M(\gamma_j)$ for the j^{th} class. This is performed using the following steps:

- As in [1], the power allocation, as a function of $\overline{\mathcal{P}}_M^e(\gamma_j)$, is given by

$$P_{k,j} = \frac{2N_k(2^{\hat{b}_{k,j}} - 1)}{3} \cdot \left[\text{erfc}^{-1} \left(\frac{\overline{\mathcal{P}}_M^e(\gamma_j)\sqrt{2^{\hat{b}_{k,j}}}}{2\left(\sqrt{2^{\hat{b}_{k,j}}} - 1\right)} \right) \right]^2 \qquad (6)$$

where $\overline{\mathcal{P}}_M^e(\gamma_j)$ and N_k are the average error probability for each class and the noise power, respectively.

IV. Simulation Results

Our results are generated assuming an application that only requires three different protection classes. Each class requires different target-rates, T_j, and a fixed noise margin step size, $\Delta\gamma$, between these classes. $\Delta\gamma$ needs to be selected according to application requirements. Here it is assumed to be fixed at 3 dB. Thus, the symbol-error rate curves would be separated by 3 dB. In our design, we also assume a 4×4 MIMO-OFDM system with $N = 2048$ subchannels (with 512 subchannels for each beam). The maximum allowed bits per subchannel used is set to $B_{\max} = 8$. In our simulation, we assumed two channels:

- Channel#1: a highly correlated channel with following diagonal $D_k = [18.66, 28\times10^{-2}, 32\times10^{-3}, 35.4\times10^{-4}]$.
- Channel#2: a highly scattered channelwith the following diagonal $D_k = [15.4, 5.56, 1.1, 0.1]$.

Figure 3 depicts the UEP bit and power loading across the assumed four MIMO eigenbeams. In this figure, λ_i represents the four channel eigenbeams. The power is allocated using (6) and bits are allocated assuming a perfect CSI.

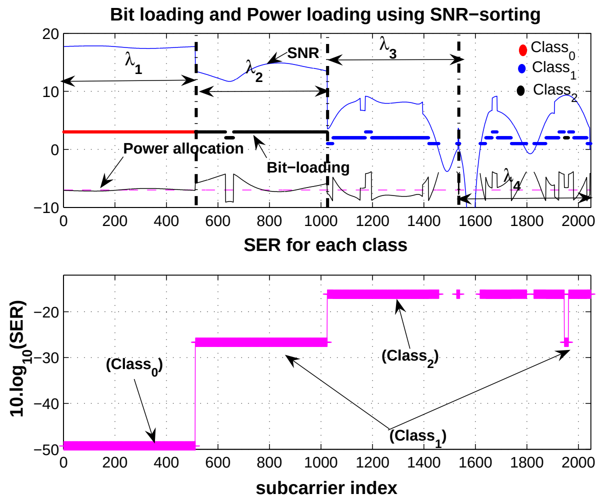

Fig. 3. UEP bit and power allocation in a 2048 subchannels MIMO-OFDM

A. Effect of Channel Feedback Error

For generatting these results, we assume an instantaneous channel with an error Ξ compared to the channel mean. The variance of this error $\sigma_\Xi^2 \in \{0, 0.001, 0.01, 0.1\}$, where $\sigma_\Xi^2 = 0$ indicates the perfect CSI. In the following results, the channel model (Channel#1) is used. As depicted in Fig. 4, it is shown that the least important data (Class 2) cannot be transmitted due to an error floor, which is the impact of the strongest eigenbeam's interference on the weakest ones.

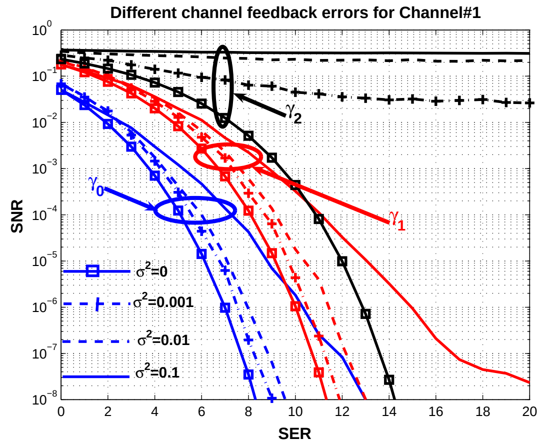

Fig. 4. Different feedback error and comparison with the perfect CSI

B. Different Beamforming Schemes

In this scheme, weaker eigenbeams are suppressed to avoid bad performance. This can realized by setting some of the eigen-vectors of the beamforming matrix $\overline{\mathbf{V}}$ to zero. Assuming the rank of the channel to be M, the order of beamforming n will be $0 \leq n \leq M$. Let us define

$$\overline{\mathbf{V}} = [\overline{\mathbf{V}}_1\overline{\mathbf{V}}_2],$$

where $\overline{\mathbf{V}}_1 = [v_1, ..., v_n]$ and $\overline{\mathbf{V}}_2 = [0, .., 0]$.

Therefore, when $n = M$, the system achieves a full-beamforming (full-BF) mode, which is not necessarily the best case. In case of a highly correlated channel matrix, the first eigenbeam is the highest one. Therefore, the interference on the next eigenbeam is maximum. In order to reduce the inteference on the eigenbeams we may go for:

1) Direct Beamforming: In this case, the columns of $\overline{\mathbf{V}}_1$ are selected to be adjacent, i.e. $\overline{\mathbf{V}}_1 = [v_1, v_2]$ for the example with $M = 4$. This will not solve the interference arising from the strongest eigenbeams, although it supresses the weaker interference.

2) Selected Beamforming: In this scheme, the columns of the beamforming matrix $\overline{\mathbf{V}}_1$ are selected to reduce interference, and they are not necessarily adjacent. In our case, we selected $\overline{\mathbf{V}}_1 = [v_1, v_3]$ for the example with $M = 4$. This choice ensures that the highly interfered eigenbeam v_2 is not selected, and its interference on the first and the third is also omitted.

As can be seen in Fig. 5, when $n = 2$ for the case of direct beamforming (BF$[v_1, v_2]$), the performance of, in case of (Channel#1), is worse than with full-BF in Fig. 4. This is due to the high interference on/from the second eigenbeam. As seen in Fig. 6, the performance of the robust method is still unacceptable using BF[v_1, v_2]. Although the performance in using BF[v_1, v_3] for both methods, robust and intuitive method, is better than BF[v_1, v_2] due to the high interference that have been suppressed by setting $v_2 = 0$. This mean that this scheme is more robust against interference from adjacent beams. It is also obvious that the robust methods works well using BF[v_1, v_3], although the intuitive method outperform the robust method. These results do not contradict the results in [1], since the robust method does not perform well in case of Gaussian noise/disturbers, and the CSI error induced interference is a zero-mean Gaussian. *Note: from the same figure, Fig. 5, it is clear that BF[v_1, v_3] does not offer a solution for highly scattered channels (Channel#2).*

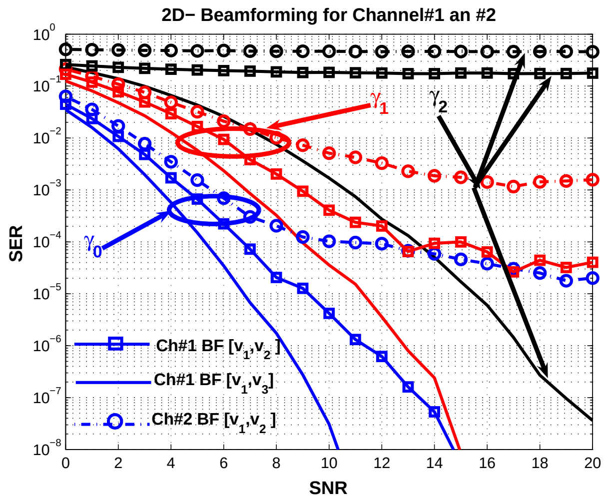

Fig. 5. 2-BF and 2-SW-BF in for the different channel and different sorting methods

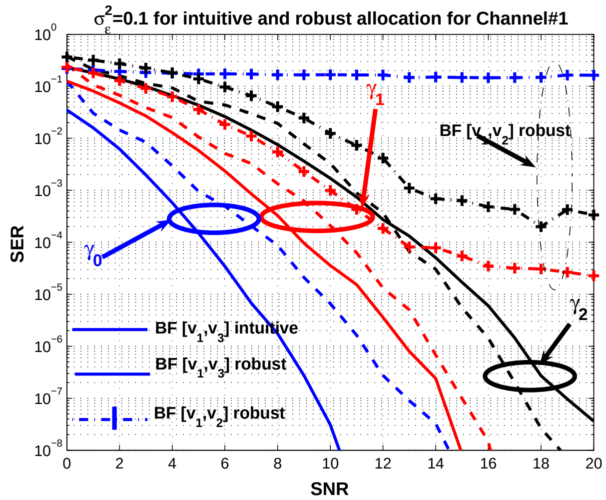

Fig. 6. Intuitive and robust allocation for the protection classes at channel feedback error of variance $\sigma_\epsilon^2 = 0.1$

V. Conclusion

We described an UEP bit-allocation scheme in the MIMO-OFDM system as a modification of an earlier non-UEP algorithm by Chow et al.. This allows arbitrary margin definitions according to bit streams of different priorities. It further allows to devote an arbitrary number of bits to these classes and allocate these classes over the given eigenbeams, which describes the bit-loading algorithm as a 2D UEP bit-loading. The intuitive method ensures higher performance due to the Gaussian distribution of the CSI error. The n-dimenssion selected beamforming can be regarded as a practical solution for supressing the interference arrises from the CSI error. Finally, it is clear that the MIMO systems size is reduced in case of partial CSI and antennas correlation.

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