

# Unequal Error Protection Multilevel Codes and Hierarchical Modulation for Multimedia Transmission

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**Abstract**—This paper presents the design of multilevel coding schemes optimized for the transport of multimedia data. The key feature is unequal error protection which allows for more protection for header information and essential data, accepting worse performance for less important payload. The approach is based on the information theoretical description of mutual information in multilevel coding schemes. We discuss the suitability and UEP capability of standard as well as non-uniform signal constellations. We investigate the flexibility of this method regarding the design freedom and verify the approach by an image transmission application.

## I. INTRODUCTION

When dealing with the transport of multimedia data, their structural properties are important for an efficient and successful transmission. Images and, in general, multimedia data often have heterogeneous error sensitivities, especially when progressive source encoding is applied. These file formats contain parts which are more important than others and, thus, detection errors due to additive noise, multipath propagation, or other disturbances on the channel may have more (or less) severe effects. The knowledge of such structural details can be utilized by protecting these different classes of importance differently during transmission in order to make it more efficient and increase the perceptible quality.

Such a heterogeneous treatment may be obtained in many ways and at different instances in a communication scheme, e.g., applying hierarchical or adaptive modulation, or adaptive bit and power allocation when using multicarrier modulation. This work, however, deals with unequal error protection (UEP) within coded modulation, especially multilevel codes (MLC). Coded modulation is a well-known technique which jointly optimizes the coding scheme and the modulation scheme [1]–[4]. The signal constellation is successively subdivided into smaller subsets, where each partitioning level is assigned a label. These labels are protected by separate channel codes with appropriate protection capabilities. The codes have to be designed carefully depending on the modulation scheme and its partitioning or labeling strategy. According to [5], the optimal way of designing the codes (in terms of mutual information) is to match the code rates to the channel capacities of the partitioning steps, assuming perfect codes. This means that, for a given signal-to-noise ratio (SNR) and given modulation scheme and partitioning, the optimal code rates are

well-defined. However, there are also other design approaches with similar results, like bit-interleaved coded modulation [6] or low-density parity-check codes optimized for a certain modulation scheme [7]. The corresponding channel codes in a multilevel coding scheme can be block codes, convolutional codes, or concatenated codes.

Previous work on unequal error protection multilevel codes has been published in [8], [9], [10], and [11]. They focus on the design of special modulation schemes for achieving unequal error protection, especially with non-uniformly spaced signal constellations where symbols are grouped and the Euclidean distances within and between those groups differ. In [8], a time-division multiplexing scheme is proposed, switching between different conventional multilevel coding schemes. In [9], non-uniform constellations together with a wavelet transform are applied in order to perform UEP image transmission. A non-uniform partitioning scheme leading to unequal error protection is presented in [10], and in [11], multiple block coded modulation is used together with nonregular partitioning. However, none of these publications focuses on the properties of the channel codes.

In this paper, we present a way to modify the original multilevel coding approach [5] in order to obtain and control unequal error protection by defining general design rules for the code dependent on a given signal constellation. We do not restrict the method to particular codes, but develop general rules which are applicable for any kind of codes. The paper is structured as follows. The system model of a multilevel coding scheme is given in Section II. Modifications for obtaining unequal error protection and their performances are given in Section III for standard signal constellations. Section IV contains a discussion about the flexibility and possible improvements of the proposed scheme. In Section V, hierarchical signal constellations are proposed to circumvent certain problems. Section VI finally verifies the construction rules by an image transmission application and conclusions are given in Section VII.

## II. SYSTEM MODEL

A multilevel code consists of an  $M$ -ary modulation scheme and a coding unit. The signal constellation is successively partitioned into subsets until the subsets only contain a single

signal point. The partitions are labeled at each partitioning level by symbols of an appropriate alphabet. A common approach for this partitioning strategy is Ungerböck's set partitioning, which maximizes the minimum Euclidean distance between any two symbols of a subset [1], [12].

The labels of each partitioning level are components of codewords of individual codes at each level. Different design strategies have been proposed for these codes. For a long time, the balanced distances rule was believed to be best suited, where  $\max \{ \min_{i=0, \dots, l-1} \{ d_i^2 \delta_i \} \}$ , and thus  $d_i^2 \delta_i = \text{const}$  holds, where  $d_i^2$  is the minimum squared Euclidean distance of the corresponding sub-constellation and  $\delta_i$  is the minimum Hamming distance of the code at level  $i$ . In [5], the authors proved the capacity design rule to be optimum in terms of mutual information. According to this rule,

$$R_i = C_i \quad (1)$$

for perfect codes, which means that the code rate at each level should be equal to the capacity of the partitioning. This is connected to Shannon's theorem which states that

- 1) a vanishing error probability is possible for  $R < C$  for block lengths going to infinity, and
- 2) the error probability will never vanish for  $R > C$ , regardless of the block length.

As a note on this, transmitting with a code rate  $R > C$  will make error-free transmission impossible, whereas a code rate  $R < C$  will just reduce efficiency but maintain the possibility of error-free transmission. In the context of finite block length codes, performance is improved the farther the rate is from capacity. Figure 1 shows the structure of such a multilevel coding transmitter with 8-PSK modulation and a 3-level set partitioning.

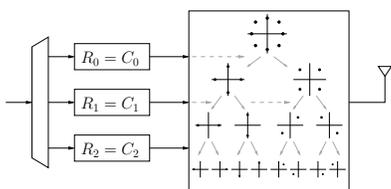


Fig. 1. Transmitter structure of an 8-PSK MLC scheme

Figure 2 shows the capacity curves for a set partitioning of an 8-PSK scheme. It contains curves for 8-PSK, QPSK, and BPSK, since the set partitioning of the 8-PSK scheme leads to these kinds of subsets. The capacities of the individual partitioning levels follow from the chain rule of mutual information, [5], and are given by

$$\begin{aligned} C_i &= I(Y; X^i | X^0 \dots X^{i-1}) \\ &= E_{x^0 \dots x^{i-1}} \{ C(\mathbf{A}(x^0 \dots x^{i-1})) \} - \\ &\quad E_{x^0 \dots x^i} \{ C(\mathbf{A}(x^0 \dots x^i)) \}, \end{aligned} \quad (2)$$

where  $C(\mathbf{A}(x^0 \dots x^{i-1}))$  represents the capacity of a signal subset  $\mathbf{A}(x^0 \dots x^{i-1})$ . The vector  $(x^0 \dots x^{l-1})$  represents the bits addressing a symbol and  $X^i$  is the random variable corresponding to the  $i$ th bit. Correspondingly,

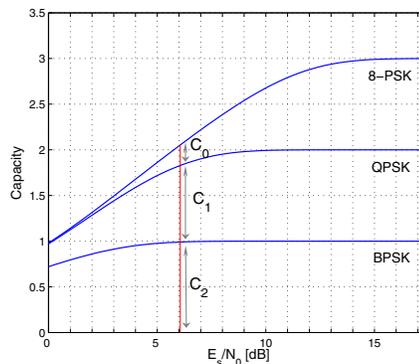


Fig. 2. Capacity curves of the partitioning levels in an 8-PSK scheme with Ungerböck's set partitioning

$Y^i$  represents the random variable of the  $i$ th bit based on the received symbol. As an example, the capacity of the first partitioning level of an 8-PSK scheme would be  $C_0 = C^{8\text{-PSK}} - C^{\text{QPSK}}$ .

A low-complex decoding method is called multistage decoding (MSD), where the levels are decoded one after another, taking into account decisions of previously decoded levels. Since the lower levels' performance is affected by the upper levels due to error propagation, the upper levels are chosen for important data and the lower ones for less important data.

The dashed lines in Fig. 4 show the error rates of the three levels in an 8-PSK multilevel code with MSD versus the SNR. Note that the SNR is given by  $E_s/N_0$  in dB in order to compare the levels in a way that shows UEP properties. The codes are Turbo codes of length  $N = 1000$  and generators  $g(D) = (1 + D + D^3 \quad 1 + D + D^2 + D^3)/(1 + D^2 + D^3)$  for both component codes. They are designed according to the capacity design rule in [5] for an operating point of  $E_s/N_0 = 6$  dB. We employ the same mother code for all levels, and different code rates are obtained by puncturing and pruning [13], [14].

### III. MODIFICATION FOR UEP

As shown in Fig. 4, the levels do not differ significantly in performance. This section describes how to modify the scheme in order to protect the bits in a symbol differently.

This work focuses on designing the coding unit rather than the modulation unit of the MLC scheme. According to the capacity design rule given in (1), the choice of the code rates is crucial to the performance of the system. Our approach is to vary the code rates in order to, on the one hand, improve the performance for the important data and, on the other hand, accept some performance degradation for less important data. In spite of the lower Euclidean distance, we assign the most important data to the first partitioning level since the other levels are affected by error propagation in case of a wrong decision in the first level in MSD.

The general idea is to allow for a lower code rate for the important data and increase the code rate of the less important data, such that the overall rate remains constant. This allows

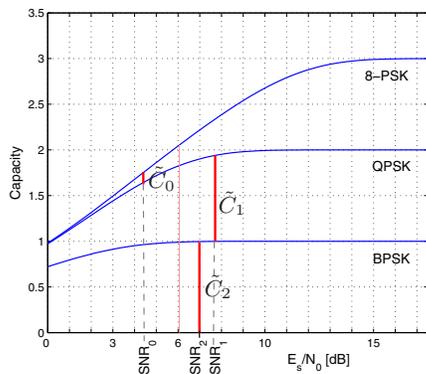


Fig. 3. Individual operation points

for a fair comparison to the original non-UEP scheme. The variation of the code rates is achieved by choosing different operating points (w.r.t. signal-to-noise ratio) for the levels. Consider again Fig. 2. For  $E_s/N_0 = 6$  dB, we have approximately the following capacities at the levels:

$$C_0 = 0.23, C_1 = 0.84, C_2 = 0.98. \quad (3)$$

Thus, the optimal system would have the code rates  $R_0 = 0.23$ ,  $R_1 = 0.84$ , and  $R_2 = 0.98$  and all levels would have their waterfall region near the overall operating point. Assume now, the first level shall be better protected. In order to accomplish this, one may choose an individual operating point for this level at a lower SNR. Considering the capacity curves in Fig. 2, this means reducing this level's capacity. Sticking to the capacity design rule, this reduces the level's optimal code rate.

In return, the code rates of the other levels have to be balanced in order to keep the overall code rate constant. How the compensation is divided onto the remaining levels has to be traded off depending on the application. Figure 3 shows the capacity curves again, now with different individual operating points. We choose  $\tilde{C}_0 = 0.09$  which means that the reduction has to be balanced by the other levels. A possible choice is

$$\tilde{C}_0 = R_0 = 0.09, \tilde{C}_1 = R_1 = 0.94, \tilde{C}_2 = R_2 = 1. \quad (4)$$

The new operating points are now located at those SNRs where the new capacities  $\tilde{C}_i$  are equal to the true capacities  $C_i$ . Thus, the individual operating points for the new capacities are approximately at 4.4 dB, 7.6 dB, and 7 dB.

The bit-error rates of the new system are shown in Fig. 4. The code parameters are the same as in Section II and the puncturing rates are designed for an operating point of  $E_s/N_0 = 6$  dB. One can see that the new waterfall regions are near the designed operating points, as desired. Changing the code rate of a level, of course, has a direct impact on the waterfall region. By designing the code rates to be equal to the true capacities at a certain operating point (SNR), all levels (should) have good performance at (and above) the operating point, and a high error rate for lower SNR. By modifying the individual code rates, each level will show this behavior

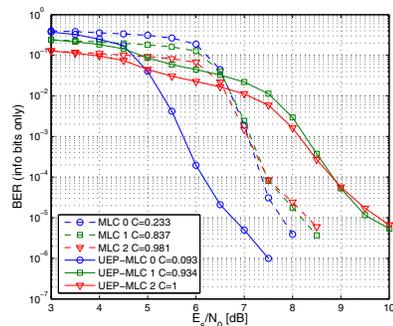


Fig. 4. Performance of an original MLC scheme and a UEP-MLC scheme with individual operating points

at that particular SNR which yields a (true) capacity equal to the code rate on that level. Note that we assume optimal codes in this consideration. The actual location of the waterfall region depends on how close the code performances are to the Shannon limit.

Furthermore, the general previous statement only holds for maximum-likelihood decoding. As shown in Fig. 3, the individual operating point of the third level may be lower than that of the second level. In the case of maximum-likelihood decoding, this would lead to waterfall regions close to these operating points. If MSD is used, the decoding process of a level is usually affected by error propagation and cannot outperform the higher levels. Thus, the resulting real operating point of level 2 would be the same as of level 1 in Fig. 3.

#### IV. FLEXIBILITY

According to the above description, the waterfall regions of a desired UEP-MLC scheme can theoretically directly be chosen by defining appropriate code rates, as long as reasonably good channel codes are used which are near to the Shannon limit. The only constraint is that the overall rate should be kept constant in order to be able to compare different codes or maintain the spectral efficiency. Generally, this method leads to a UEP system which is very easy to design and control.

However, taking the capacity curves with set partitioning into consideration, the choice of code rates can be very limited when the overall rate should be maintained. There is only a small SNR range where it is possible to trade off the code rates. Consider Fig. 2 again and imagine a desired overall operating point of 11 dB. The capacities of both the second and the third level are 1. Thus, the code rate of the first level can not be reduced without reducing the overall code rate. Generally, the more levels have capacities smaller than 1, the larger the degree of freedom in the design of a UEP scheme. In contrast, at an operating point of 4 dB, the first level already has a very low rate and it does not make much sense to reduce it even further, in favor of data throughput.

To circumvent the problem at high SNR, where  $R_i = 1$ ,  $0 < i \leq l - 1$ , one can still reduce the code rate of the first level. In order to maintain the throughput, an appropriate

amount of information data from the last, least important level can be omitted before encoding which, in fact, leads to a rate  $R_{l-1} > 1$  and allows  $R_0$  to be reduced. In the context of scalable multimedia data, where UEP is desired, truncation of the least important data in favor of more important data is a common method. Another solution to the problem are hierarchical signal constellations which are presented in the next section.

## V. HIERARCHICAL SIGNAL CONSTELLATIONS

In this section, we present UEP-MLC codes in combination with hierarchical signal constellations which may be designed to provide a desired amount of UEP. In [15], [16], hierarchical modulation is presented, where the symbols are not uniformly distributed in the signal space but are basically grouped such that the single bit positions in a symbol have different (or even desired) error probabilities. Figure 5 shows an exemplary hierarchical 16-QAM signal constellation with inter-symbol distances  $d_0$  and  $d_1$ . By optimizing these distances, or their relation  $d_0/d_1$ , the amount of UEP can be varied. However, small losses in the channel capacity have to be accepted for certain SNR regions.

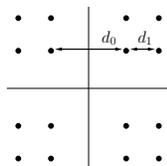


Fig. 5. Hierarchical 16-QAM signal constellation

Note that this 16-QAM scheme can be seen as an outer and an inner QPSK alphabet. A goal of the constellation design should be that the capacities of the partitioning levels should be more uniform than for standard signal constellations. Especially, the capacity of the first (and second) level should be increased such that shifting operating points is feasible. The labeling or partitioning strategy for this example is chosen such that the first two bits are used to address the 'clouds' of the outer QPSK scheme and the last two bits address a symbol in the respective cloud. The further the clouds lie apart, the more protected are the first two bits. This also means that the capacity of the first two bits is increased with the relation  $d_0/d_1$ , while the capacities of the other two bits is decreased. Figure 6 shows the capacities of the partitioning levels of a uniform 16-QAM scheme with set partitioning and a hierarchical 16-QAM scheme with  $d_0/d_1 = 2$ . It is clear that the overall channel capacity of the hierarchical modulation is slightly lower than that of the standard modulation for some SNRs. However, the curves are much more suitable for UEP-MLC than standard modulation, because the distances between the capacities are more uniform and shifting individual operating points is feasible for a much bigger SNR region. Assuming an operating point of  $E_s/N_0 = 11$  dB, the capacities are

$$C_0 = 0.98, C_1 = 0.98, C_2 = 0.56, C_3 = 0.56. \quad (5)$$

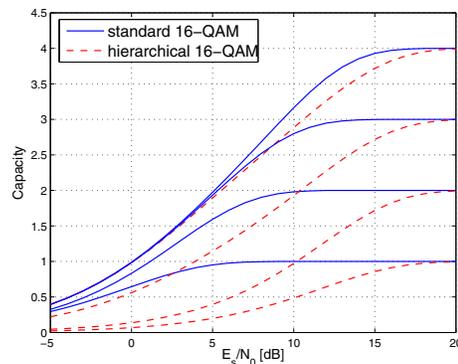


Fig. 6. Capacity curves of the partitioning levels in a standard 16-QAM scheme with Ungerböck's set partitioning and a hierarchical 16-QAM scheme with  $d_0/d_1 = 2$

When shifting the individual operating points, the SNR range where this is sensible and efficient is much larger and the difference in protection between high and low protection classes may be chosen much larger.

Generally, a higher flexibility in the choice of operating points can already be achieved with Gray labeling instead of set partitioning to some extent. However, hierarchical modulation offers an even higher degree of freedom and flexibility. It does not only make UEP design feasible but in a way controllable by optimizing the factor  $d_0/d_1$ , or even designing this factor differently for real and imaginary signal components.

It should be noted that hierarchical modulation together with conventional MLC (without individual operating points) is not useful because the capacity design rule will 'destroy' the UEP from modulation. Only when creating individual operating points, one will gain from hierarchical modulation.

## VI. IMAGE TRANSMISSION APPLICATION

In this section, we show two UEP-MLC examples of the above solution. First, an image is divided into smaller blocks of size  $8 \times 8$  bytes which are (each) progressively source-encoded using the *Embedded Zero-Tree Wavelet* (EZW) algorithm. The algorithm forces the data to be arranged according to their importance, with the most important data first. Afterwards, the sequence of source-encoded blocks is assigned to the levels of a UEP-MLC scheme, the most important data are encoded on the first level and so on, and such that the codewords are of equal size on all levels. This means that the fraction of bits assigned to the first level is  $R_0/\log_2(M)$ , and accordingly for the following levels, where  $\log_2(\cdot)$  is the dual logarithm. After modulation, the symbols are transmitted over an AWGN channel, decoded using MSD and fed to the source-decoder. In case the receiver detects an error in a block (which can easily be done by an additional error detecting code), the source-decoder only considers data from the beginning of the block up to the error location and omits the rest.

We choose an overall operating point of  $E_s/N_0 = 11$  dB. According to the shown bit-error rates of a general MLC scheme, the error rates will still be high at exactly the operating point

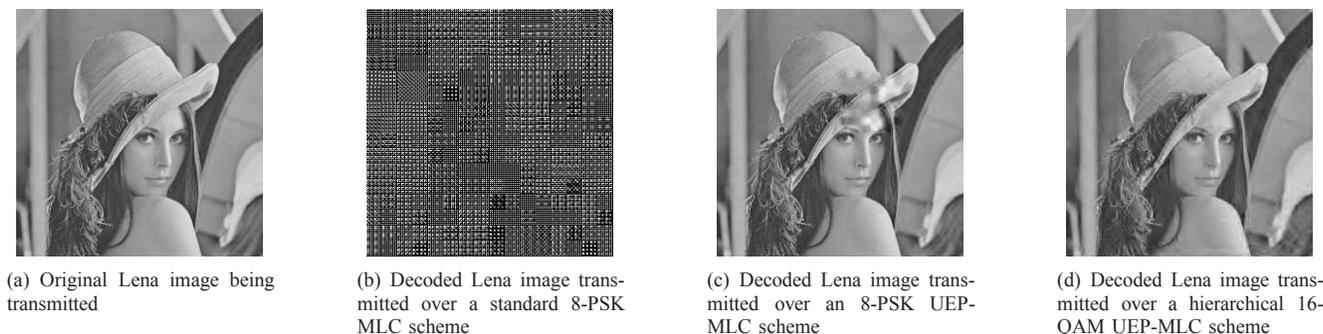


Fig. 7. Original image (a), transmitted over a conventional 8-PSK MLC scheme (b), over an 8-PSK UEP-MLC scheme (c), and over a hierarchical 16-QAM UEP-MLC scheme (d) at  $E_s/N_0 = 11$  dB.

but start to drop immediately. This means bad quality at the receiver side. We verify this by transmitting the Lena image over a conventional MLC system with 8-PSK at  $E_s/N_0 = 11$  dB. The codes are designed for an overall operating point exactly at this SNR, i.e.,  $R_0 = 0.8$ ,  $R_1 = 1$ ,  $R_2 = 1$ . Figures 7(a) and 7(b) show the original image and the received and decoded image. As expected, the quality is bad and the image is not identifiable. In order to account for the structural properties of the image file, we modify the system to provide UEP by defining the code rates to be  $R_0 = 0.375$ ,  $R_1 = 1$ ,  $R_2 = 1.425$ , which corresponds to individual operating points of 7.5 dB and 10.5 dB for levels 0 and 1. For level 3, the code rate does not have an equivalent operating point, since the capacity cannot exceed 1. Note that the code rate  $R_2 = 1.4125$  is, in fact, accomplished by applying no code at all and omitting the last 29.82% of the third level's information sequence. As codes, again Turbo codes were used together with puncturing and pruning. Figure 7(c) shows the resulting image which contains only a few errors in the whole picture. A similar simulation was done with a 16-QAM scheme. Figure 7(d) shows the decoded image after transmission with hierarchical 16-QAM modulation with  $d_0/d_1 = 2$ , where the operating points were chosen to be 2 dB, 3 dB, 17.5 dB, and 15 dB. In this case, even the few errors which are still present in Fig. 7(c) were able to be corrected.

## VII. CONCLUSIONS

We have designed an unequal error protection multilevel coding scheme where the waterfall regions of the different protection levels can directly be chosen by trading the code rates of the partitioning levels. The application of non-uniform signal constellations further improves the flexibility of the design compared to standard signal constellations. When dealing with progressively encoded source files, truncating least important information bits makes the approach even more flexible, which is a common approach in the context of scalable data processing. Therewith, we have designed a flexible and very easy-to-control UEP-MLC scheme.

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