A Receiver Design for MIMO Systems over Rayleigh Fading Channels with Correlated Impulse Noise

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***Abstract*—A Middleton Class-A (MCA) density is well known to model impulsive interference. The statistical-physical extension of this model for multiple receive antennas is currently limited to two antennas. An algebraic extension of the univariate MCA model leads to a multivariate MCA distribution, which can be used for an arbitrary number of receive antennas. Since recent studies show a significant level of noise correlation in several wireless systems, we develop MIMO receivers for Rayleigh fading channels in the presence of spatially correlated MCA interference. We derive an upper bound pairwise error probability (PEP) for orthogonal space time block codes (OSTBCs). We show that the performance improvement of OSTBCs is highly dependant on the impulse noise environment and it becomes minor as the number of transmit and receive antennas increases. In the design of MIMO receivers, the maximum likelihood (ML) detection has a high computational complexity. Since the MCA model can be seen as a multivariate Gaussian distribution conditioned on the knowledge of noise state, we introduce a simple approach to estimate the state of noise at the receiver, which subsequently reduces the complexity of the ML decision rule.**

***Index Terms*—impulsive interference, Middleton Class-A density, pairwise error probablility, MIMO system.**

I. Introduction

Impulsive interference corrupts a variety of practical wireless systems such as radio frequency interference (RFI) in indoor and outdoor channels [1], [2], RFI generated by computers in embedded wireless data transceivers [3], and co-channel interference in a Poisson field of interferers [4]. The source of interference can be either natural or man-made such as atmospheric noise, power lines, ignition, and closely located wireless systems.

Middleton's Class-A (MCA) model [2] is one of the most accepted statistical-physical models for impulsive interference superimposed to additive white Gaussian noise (AWGN). This model is characterized by two basic parameters that can be adapted to fit a wide variety of impulse noise phenomena occurring in practice. For multiple antenna systems, the statistical-physical extension of this model is derived for two closely-spaced antennas under the assumption of narrowband and far-field interference [5]. Extending the Middleton model for more receive antennas is complicated, which restricts the analysis to two receive antennas. For an arbitrary number of receive antennas, the algebraic extension is found to dissolve the statistical-physical modeling limitations, which leads to a multivariate MCA model [6], [7].

The research into investigating the effect of MCA noise on MIMO performance is not extensive [6]. The effect of noise correlation for MIMO systems is considered in [8]. However, its analysis is limited to two receive antennas. The results of [6] showed a gain loss in the performance of orthogonal space time codes (OSTBs) in uncorrelated MCA channels. So far, it is not clear how the code behaves in case of correlated MCA channels, which motivates us to derive the general formula for an upper bound of the pairwise error probability (PEP) in multivariate MCA channels. Thereafter, we proceed to the MIMO receiver design for the considered channels to reduce the complexity of a maximum likelihood (ML) detector.

This paper is organized as follows: Section II briefly describes the system model and the multivariate MCA model for spatially correlated interference. In Section III, we introduce the upper bound PEP derivation of OSTBC in the presence of correlated and uncorrelated MCA noise. In Section IV, the design problem of a MIMO receiver is considered. Finally, simulation results and concluding remarks are presented in sections V and VI, respectively.

II. System Model

We consider a MIMO wireless communication channel where the transmitter and receiver are equipped with M_t and M_r antennas, respectively. We further assume that the transmitter employs an ST coding scheme, where the vector of M_b information symbols $\mathrm{s} = [s_1 s_2 \cdots s_{M_b}]^T$, which are taken from complex signal constellations such as PSK or QAM, is encoded into a ST code matrix of size $M_t \times T_b$ as follows:

$$\mathbf{X} = \begin{pmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,T_b} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,T_b} \\ \vdots & \vdots & \ddots & \vdots \\ x_{M_t,1} & x_{M_t,2} & \cdots & x_{M_t,T_b} \end{pmatrix}, \qquad (1)$$

where the entries $x_{i,t}$ denote the coded symbols transmitted from the i^{th} transmit antenna at time instant t, for $1 \le i \le M_t$, $1 \le t \le T_b$, where T_b is the block length. The channel is assumed to remain constant during the block length. Therefore, the received signal at the M_r receive antennas can be expressed as

$$\mathbf{Y} = \sqrt{\frac{E_s}{N_0 M_t}}\mathbf{H}\mathbf{X} + \mathbf{Z} \,, \tag{2}$$

where $\mathbf{Y} \in \mathbb{C}^{M_r \times T_b}$ is the received signal matrix, E_s is the transmitted energy per symbol, $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ is the MIMO channel matrix of independent and identical distributed (i.i.d.) complex Gaussian entries with zero mean and unit variance, N_0 is the noise variance, and $\mathbf{Z} \in \mathbb{C}^{M_r \times T_b}$ denotes the additive complex MCA noise matrix.

The entries of $\mathbf{Z}$, $z_{j,t}$, denote the samples of a complex MCA noise process corrupting the signal at the j^{th} receive antenna at time instant t. The interference process as seen by the j^{th} receive antenna includes two noise components: a Gaussian component $n_{j,t}$, which describes the thermal background noise generated at the receiver and an impulsive component $w_{j,t}$ due to the interference from various man-made or natural sources. Hence, the received noise at the j^{th} antenna is given by

$$z_{j,t} = n_{j,t} + w_{j,t} \,, \tag{3}$$

where $n_{j,t}$ and $w_{j,t}$ are assumed to be statistically independent. The baseband noise samples $z_{j,t}$ can be modeled by a complex MCA density as in [7]

$$p(z_{j,t}) = \sum_{m=0}^{\infty} \frac{\alpha_m}{\pi\sigma_{m,j}^2} e^{-\frac{|z_{j,t}|^2}{\sigma_{m,j}^2}} \,, \tag{4}$$

where

$$\alpha_m = \frac{A^m e^{-A}}{m!} \,, \tag{5}$$

and $\sigma_{m,j}^2 = \frac{m/A+\Gamma_j}{1+\Gamma_j}$. The impulsive index, A, is the product of the average number of impulses per second with their mean duration. The Gaussian factor, $\Gamma_j = \mathrm{var}[n_{j,t}]/\mathrm{var}[w_{j,t}]$, represents the power ratio of the Gaussian component $n_{j,t}$ to the impulsive component $w_{j,t}$ at the j^{th} receive antenna. The specified range of A and Γ_j are within $[10^{-2}\,1]$ and $[10^{-6}\,1]$, respectively. Note that (4) reduces to a Gaussian density when $A \to \infty$. The MCA density can be seen as a Gaussian distribution conditioned on the values of m, where m represents the noise state. According to (5), the noise state m is a Poisson-distributed random variable such that the probability of being in a given state is equal to α_m. Moreover, for a given noise state, m, we can indicate that there is no impulse, i.e., $m = 0$, or, impulses are present, i.e., $m \geq 1$.

Since the M_r receive antennas are subject to the same physical process creating the impulse, at time instant t, the elements of the received noise vector $\mathbf{z}_t = [z_{1,t} \cdots z_{M_r,t}]^T$ can be assumed jointly dependent. Therefore, a complex multivariate MCA model can be used to model $\mathbf{z}_t$ as follows:

$$p(\mathbf{z}_t = [z_{1,t} \cdots z_{M_r,t}]) = \sum_{m=0}^{\infty} \alpha_m g(\mathbf{z}_t, \mathbf{\Sigma}_m) \,, \tag{6}$$

where

$$g(\mathbf{z}, \mathbf{\Sigma}) = \frac{1}{\pi^{M_r}|\mathbf{\Sigma}|} e^{-\mathbf{z}^H \mathbf{\Sigma}^{-1}\mathbf{z}} \,, \tag{7}$$

$|\cdot|$ denotes a determinant, and $(\cdot)^H$ represents the Hermitian transpose. $\mathbf{\Sigma}_m$ is the covariance matrix, which has the following form

$$\mathbf{\Sigma}_m = \begin{pmatrix} \sigma_{m,1}^2 & \cdots & \rho_{1M_r}^m \sigma_{m,1}\sigma_{m,M_r} \\ \vdots & \ddots & \vdots \\ \rho_{M_r1}^m \sigma_{m,M_r}\sigma_{m,1} & \cdots & \sigma_{m,M_r}^2 \end{pmatrix} \,, \tag{8}$$

where ρ_{jk}^m is the correlation coefficient of the noise samples at the j^{th} and k^{th} receive antennas for a noise state m. The covariance of noise samples between the j^{th} and k^{th} receive antennas is given by

$$\begin{aligned} \mathrm{E}[z_{j,t}^* z_{k,t}] &= \int_{\mathbb{C}_j}\int_{\mathbb{C}_k} z_{j,t}^* z_{k,t} p\left(\mathbf{z}_t = [z_{j,t}\ z_{k,t}]\right) \mathrm{d}z_j \mathrm{d}z_k \\ &= \sum_{m=0}^{\infty} \alpha_m \sigma_{m,j}\sigma_{m,k}\rho_{jk}^m \,, \end{aligned} \tag{9}$$

where $\mathbb{C}_j$ and $\mathbb{C}_k$ are the complex planes for $z_{j,t}$ and $z_{k,t}$. From (9), It is clear that $\mathrm{E}\left(|z_{j,t}|^2\right) = 1$. Therefore, N_0, that appears in (2), controls the noise variance at the receiver. Thermal noise is more uncorrelated than impulse noise, this would result in correlation coefficients being dependent on m. This property is discussed in (6), where the correlation coefficients ρ_{jk}^m are assumed to not necessarily be identical for all noise states.

For two receive antennas, the proposed model (6) can be seen as a complex extension of a bivariate MCA model [5], which has been derived through a statistical-physical modeling. Under the assumptions of uncorrelated noise observations with equal Gaussian factors, we have $\mathbf{\Sigma}_m = \sigma_m^2 \mathbf{I}_{M_r}$, where $\sigma_{m,j}^2 = \sigma_m^2, \forall\, j$. Hence, (6) reduces to a balanced MCA model considered in [6], [7].

To specify the present model over the length of the STBC, we assume that the noise vectors $\mathbf{z}_t$, $\forall t = 1, \cdots, T_b$ are statistically independent. It follows that the noise matrix $\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_{T_b}]$ has the following density

$$p(\mathbf{Z}) = \prod_{t=1}^{T_b}\sum_{m=0}^{\infty} \frac{\alpha_m}{\pi^{M_r}|\mathbf{\Sigma}_m|} e^{-\mathbf{z}_t^H \mathbf{\Sigma}_m^{-1}\mathbf{z}_t} \,. \tag{10}$$

III. Code Design Criteria

In this section, we evaluate the ST code criteria in the presence of correlated MCA noise. We assume that the channel matrix $\mathbf{H}$ is known at the receiver. In addition, we assume that the noise states m_t, $\forall t = 1, \cdots, T_b$, are also known at the receiver [7]. Alhough this assumption is not realistic, in Section IV, we introduce a simple approach to realize this assumption at the receiver.

According to (2), we stack all T_b columns of the received martix $\mathbf{Y}$ into a single column vector. Then, the received signal can be written as

$$\mathbf{y} = \sqrt{\frac{E_s}{N_0 M_t}}(\mathbf{X}^T \otimes \mathbf{I}_{M_r})\mathbf{h} + \mathbf{z} \,, \tag{11}$$

where $\mathbf{I}_n$ denotes the identity matrix of size n, $\mathbf{X}$ is as given in (1) and $\otimes$ is the Kronecker product. Under the assumption that the noise states are available at the receiver, the vector $\mathbf{z} \in \mathbb{C}^{M_r T_b \times 1}$ can be seen as a conditional complex multivariate Gaussian vector with the positive definite covariance matrix

$$\boldsymbol{\Sigma} = \mathrm{E}\left[\mathbf{z}\mathbf{z}^H\right] = \begin{pmatrix} \boldsymbol{\Sigma}_{m_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{m_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Sigma}_{m_{T_b}} \end{pmatrix} , \quad (12)$$

where $\boldsymbol{\Sigma}_m$ is given in (8). Like a conventional system when the received signals are impaired by correlated Gaussian noise, a noise-whitening matrix can be applied to the received signals to obtain equivalent samples with uncorrelated noise. In the case of correlated MCA noise, the inverse of $\boldsymbol{\Sigma}$ can be factorized as $\boldsymbol{\Sigma}^{-1} = \mathbf{L}\mathbf{R} = \mathbf{L}\mathbf{L}^H$. Multiplying (11) by $\mathbf{L}^H$, we obtain

$$\mathbf{L}^H\mathbf{y} = \sqrt{\frac{E_s}{N_0 M_t}}\mathbf{L}^H(\mathbf{X}^T \otimes \mathbf{I}_{M_r})\mathbf{h} + \mathbf{L}^H\mathbf{z} \,. \quad (13)$$

Now, the elements of the noise vector $\mathbf{L}^H\mathbf{z}$ are i.i.d. Gaussian distributed random variables of unit variance. Herewith, the optimum decision rule can be expressed as

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \left|\mathbf{L}^H\mathbf{y} - \sqrt{\tfrac{E_s}{N_0 M_t}}\mathbf{L}^H(\mathbf{X}^T \otimes \mathbf{I}_{M_r})\mathbf{h}\right|^2 . \quad (14)$$

Assume that $\mathbf{X}_0$ is the transmitted ST matrix and the receiver can decide between two ST matrices $\mathbf{X}_0$ and $\mathbf{X}_1$. Denote $\mathbf{S}_0 = \mathbf{X}_0^T \otimes \mathbf{I}_{M_r}$ and $\mathbf{S}_1 = \mathbf{X}_1^T \otimes \mathbf{I}_{M_r}$. The pairwise error probability (PEP) that $\mathbf{X}_0$ was sent and $\mathbf{X}_1$ is detected can be expressed as

$$P(\mathbf{X}_0 \to \mathbf{X}_1|\mathbf{h}, m_t) = Q\left(\sqrt{\tfrac{d^2_{\mathbf{h},m_t}(\mathbf{X}_0,\mathbf{X}_1)}{2}}\right) , \quad (15)$$

where $Q(x) = \frac{1}{2}\mathrm{erfc}(\frac{x}{\sqrt{2}})$ and

$$d^2_{\mathbf{h},m_t}(\mathbf{X}_0, \mathbf{X}_1) = \frac{E_s}{N_0 M_t}\mathbf{h}^H\mathbf{B}\mathbf{h} \,, \quad (16)$$

where $\mathbf{B} = (\mathbf{S}_0 - \mathbf{S}_1)^H\boldsymbol{\Sigma}^{-1}(\mathbf{S}_0 - \mathbf{S}_1)$ is the code distance matrix. Since $\mathbf{B}$ is Hermitian, there exists a unitary matrix $\mathbf{V}$, i.e., $\mathbf{V}^H\mathbf{V} = \mathbf{I}$, such that $\mathbf{V}^H\mathbf{B}\mathbf{V} = \boldsymbol{\Delta}$, where $\boldsymbol{\Delta}$ is a diagonal matrix with diagonal elements $\lambda_{(i-1)M_r+j}$, for $1 \leq i \leq M_t$ and $1 \leq j \leq M_r$, the eigenvalues of $\mathbf{B}$. Therefore, (16) reduces to

$$d^2_{\mathbf{h},m_t} = \frac{E_s}{N_0 M_t}\sum_{i=1}^{M_t}\sum_{j=1}^{M_r}\lambda_{(j-1)M_t+i}|h_{j,i}|^2 \,. \quad (17)$$

Using the Chernoff bound, (15) can be upper bounded as

$$P(\mathbf{X}_0 \to \mathbf{X}_1|\mathbf{h}, m_t) \leq \frac{1}{2}\exp\left(-\tfrac{d^2_{\mathbf{h},m_t}(\mathbf{X}_0,\mathbf{X}_1)}{4}\right) . \quad (18)$$

Since the channel coefficients $h_{j,i}$ are i.i.d. Gaussian distributed random variables, by averaging (18) with respect to $|h_{j,i}|$, we obtain

$$P(\mathbf{X}_0 \to \mathbf{X}_1|m_t) \leq \frac{1}{2}\prod_{i=1}^{M_t}\prod_{j=1}^{M_r}\left(\tfrac{1}{1+\frac{E_s}{4N_0M_t}\lambda_{(i-1)M_r+j}}\right) . \quad (19)$$

For OSTBC [9], we consider the square code matrix $\mathbf{X}$ that satisfies the orthogonality conditions, i.e., $\mathbf{X}\mathbf{X}^H = \sum_{i=1}^{M_b}|s_i|^2\mathbf{I}_{M_t}$ and $M_t = T_b$. The code distance matrix $\mathbf{B}$ has a full rank of $M_r M_t$. Since $\mathbf{B} = ((\mathbf{X}_0 - \mathbf{X}_1)^T \otimes \mathbf{I}_{M_r})^H\boldsymbol{\Sigma}^{-1}((\mathbf{X}_0 - \mathbf{X}_1)^T \otimes \mathbf{I}_{M_r})$, the eigenvalues of $\mathbf{B}$ can be expressed as $\lambda_{(i-1)M_r+j} = \beta_i\zeta_{(i-1)M_r+j}$, where β_i and $\zeta_{(i-1)M_r+j}$ are the eigenvalues of $(\mathbf{X}_0 - \mathbf{X}_1)(\mathbf{X}_0 - \mathbf{X}_1)^H$ and $\boldsymbol{\Sigma}^{-1}$, respectively. Since $\mathbf{X}$ is a square matrix, the index i in $\zeta_{(i-1)M_r+j}$, $1 \leq i \leq M_t$, is equivalent to the index t in (12) $\forall t = 1, \cdots, M_t$. As the noise state probability is given by (5), taking the expectation with respect to m_i, the upper bound of PEP reduces to

$$\begin{aligned} P(\mathbf{X}_0 \to \mathbf{X}_1) &\leq \frac{1}{2}\sum_{m_i=0}^{\infty}\alpha_{m_i}\prod_{i=1}^{M_t}\prod_{j=1}^{M_r}\left(\tfrac{1}{1+\frac{E_s\beta_i}{4N_0M_t}\zeta_{(i-1)M_r+j}}\right) \\ &\leq \frac{1}{2}\mathrm{E}_{m_i}\left(\textstyle\prod_{i=1}^{M_t}\prod_{j=1}^{M_r}\left(\tfrac{1}{1+\frac{E_s\beta_i}{4N_0M_t}\zeta_{(i-1)M_r+j}}\right)\right) , \end{aligned} \quad (20)$$

where $\mathrm{E}_m(\cdot)$ denotes the expectation with respect to m. Since $\boldsymbol{\Sigma}_{m_t}$ are independent at different time slots, it follows that $\zeta_{(i-1)M_r+j}$ are also independent for different i. Then we obtain

$$P(\mathbf{X}_0 \to \mathbf{X}_1) \leq \frac{1}{2}\prod_{i=1}^{M_t}\mathrm{E}_m\left(\textstyle\prod_{j=1}^{M_r}\left(\tfrac{1}{1+\frac{E_s\beta_i}{4N_0M_t}\zeta_{(i-1)M_r+j}}\right)\right) . \quad (21)$$

At high signal-to-noise ratios, the right-hand side of (20) can be approximated as

$$P(\mathbf{X}_0 \to \mathbf{X}_1) \leq \left(\frac{E_s}{4N_0M_t}\right)^{-M_rM_t}\left(\prod_{i=1}^{M_t}\beta_i\right)^{-M_r} \times \prod_{i=1}^{M_t}\left(\mathrm{E}_m\left(\textstyle\prod_{j=1}^{M_r}\tfrac{1}{\zeta_{(i-1)M_r+j}}\right)\right) . \quad (22)$$

We note that the first two terms of (22) are exactly the upper bound of the PEP in the case of uncorrelated Gaussian channels [10]. The last term of (22) can be seen as a performance loss factor due to the dependency of noise observations at the receive antennas. Denote the performance loss factor as

$$P_{\mathrm{loss}} = \prod_{i=1}^{M_t}\left(\mathrm{E}_m\left(\textstyle\prod_{j=1}^{M_r}\tfrac{1}{\zeta_{(i-1)M_r+j}}\right)\right) . \quad (23)$$

We note that P_{loss} depends on the numbers of transmit and receive antennas and on the eigenvalues of $\boldsymbol{\Sigma}^{-1}$, which are related to the noise statistics, i.e., A, m, Γ_j, and $\rho^m_{j,k}$.

For spatially correlated Gaussian noise, the channel has a one noise state, i.e., $m_t = 0$, $\forall t$, then $\boldsymbol{\Sigma}_{m_t}$ is equal to $\boldsymbol{\Sigma}_0$, $\forall t$. Therefore, (12) can be expressed as $\boldsymbol{\Sigma}^{-1} = \mathbf{I}_{M_t} \otimes \boldsymbol{\Sigma}_0^{-1}$ with eigenvalues $\zeta_{(i-1)M_r+j} = \zeta_j$, $\forall i, j$, where ζ_j are the eigenvalues of $\boldsymbol{\Sigma}_0^{-1}$. P_{loss} reduces to

$$P_{\mathrm{loss}} = \left(\prod_{j=1}^{M_r}\frac{1}{\zeta_j}\right)^{M_t} . \quad (24)$$

Since ζ_j are related to Γ_j and ρ_{jk}^0, then $P_{\rm loss}$ no longer depends on A. From (23) and (24), we can state that in correlated noise environments (Gaussian or impulsive) the performance of OSTBC is highly dependent on the numbers of transmit and receive antennas. Moreover, in impulse noise, the performance gain of OSTBC is varied with respect to noise statistics. To clarify this point, we assume uncorrelated impulse noise, i.e., $\rho_{jk}^m = 0, \forall j,k$. Then, (12) is a diagonal matix, i.e., $\mathbf{\Sigma}_{m_i} = {\rm diag}[\sigma^2_{m_i,1}, \cdots, \sigma^2_{m_i,M_r}]$. The eigenvalues of $\mathbf{\Sigma}^{-1}$ can be given as

$$\zeta_{(i-1)M_r+j} = \frac{1}{\sigma^2_{m_i,j}} = \frac{1+\Gamma_j}{\frac{m_i}{A}+\Gamma_j} \ . \tag{25}$$

Substituting (25) in (23), we obtain

$$P_{\rm loss} = \left({\rm E}_m \left(\prod_{j=1}^{M_r} \frac{\frac{m}{A}+\Gamma_j}{1+\Gamma_j} \right) \right)^{M_t} \ . \tag{26}$$

When $M_r = 2$, (26) can be rewritten as

$$P_{\rm loss} = \left(1 + \frac{1}{A(1+\Gamma_1)(1+\Gamma_2)} \right)^{M_t} \ . \tag{27}$$

From (27), we can describe the performance loss of OSTBC. As $A \to \infty$ (Gaussian case), (27) goes to 1, the OSTBC operates with the maximum diversity gain. As $A \to 10^{-2}$ (strongly impulsive channel), the performance loss factor is increased to high values, and subsequently, the OSTBC will not offer a substantial improvement as in the Gaussian case. For $M_r = 1$, ${\rm E}_m(\sigma^2_{m,j}) = 1$, we can see that the diversity order can be kept maximum for impulse noise. This case can be realized by empolying either time diversity [11] or ST codes with $M_r = 1$.

IV. Receiver Design

In this section, we will proceed with the receiver design of MIMO systems in the presence of MCA noise. In the following analysis, we assume that the MIMO channel matrix $\mathbf{H}$ is known at the receiver. Moreover, we assume that the noise statistics A and Γ_j, $\forall j$, are perfectly estimated from the received samples [12].

A. Minimum Distance Receiver

OSTBCs are designed according to diversity criteria that guarantee a maximum diversity gain of order $M_t M_r$ [9]. The ML decoding of OSTCs over AWGN leads to an MDR, which minimizes the Euclidean distance. The MDR over impulsive channels assumes that the incoming noise is Gaussian. Hence, it ignors the impulsive parts of (14), i.e., $m \geq 1$, which leads to

$$\hat{\mathbf{X}}_{\rm MDR} = \arg\min_{\mathbf{X}} \sum_{t=1}^{T_b} \left| \mathbf{L}_0^H \mathbf{y}_t - \sqrt{\tfrac{E_s}{N_0 M_t}} \mathbf{L}_0{}^H \mathbf{H} \mathbf{x}_t \right|^2 \ , \tag{28}$$

where $\mathbf{\Sigma}_m^{-1} = \mathbf{L}_m \mathbf{L}_m^H$, $\forall m$, and $\mathbf{x}_t$ denotes the $t^{\rm th}$ column of $\mathbf{X}$. Since $\hat{\mathbf{X}}_{\rm MDR}$ is optimum when the noise is Gaussian, it becomes suboptimum in an impulsive noise channel.

B. Optimum Receiver

Deriving the ML receiver for OSTBC in the presence of MCA noise is considered in [6], [8]. The ML receiver uses the following detection rule [6]

$$\hat{\mathbf{X}}_{\rm ML} = \arg\max_{\mathbf{X}} \prod_{t=1}^{T_b} \sum_{m=0}^{\infty} \alpha_m g\left(\mathbf{y}_t - \sqrt{\tfrac{E_s}{N_0 M_t}} \mathbf{H}\mathbf{x}_t, \mathbf{\Sigma}_m \right) \ , \tag{29}$$

where $g(\cdot,\cdot)$ is given in (7). It was shown in [6], [8] that the infinite sum in the MCA density may be truncated to two terms only, still maintaining the same performance. The optimum metric (29) reduces to

$$\hat{\mathbf{X}}_{\rm ML} \approx \arg\max_{\mathbf{X}} \prod_{t=1}^{T_b} \left[\alpha_0 g\left(\mathbf{y}_t - \sqrt{\tfrac{E_s}{N_0 M_t}} \mathbf{H}\mathbf{x}_t, \mathbf{\Sigma}_0 \right) + \alpha_1 g\left(\mathbf{y}_t - \sqrt{\tfrac{E_s}{N_0 M_t}} \mathbf{H}\mathbf{x}_t, \mathbf{\Sigma}_1 \right) \right] \ . \tag{30}$$

where $\alpha_0 = {\rm e}^{-A}$ and $\alpha_1 = 1 - {\rm e}^{-A}$, which represent the noise state probabilities of the Gaussian state and the impulsive one, respectively. As we can see in (29) and (30), the computational complexity of the ML receiver in MCA noise is significantly higher than its Gaussian noise counterpart, since the likelihood functions given by (29) and (30) can no longer be expressed as a form of MDR.

C. Approximated ML Receiver

We showed in Section III that when the noise states m_t, $\forall t = 1, \cdots T_b$, are available at the receiver, the ML rule can be simplified to an MDR. Since this assumption is not realistic [7], in this section, we introduce a simple approach to estimate the noise states and subsequently reduce the complexity of the ML receiver. Here, we start with the two-term representation of the multivariate MCA noise model, which can be expressed as

$$p(\mathbf{z}_t) = \alpha_0 g(\mathbf{z}_t, \mathbf{\Sigma}_0) + \alpha_1 g(\mathbf{z}_t, \mathbf{\Sigma}_1) \ . \tag{31}$$

Since the second term of (31) represents the impulsive events, the tails of this term are heavier than those of the first term. Therefore, by evaluating the boundary when the two terms are equal, we can distinguish between Gaussian samples and impulsive ones. The noise density can be further approximated as

$$p(\mathbf{z}_t) \approx \begin{cases} \overbrace{\alpha_0 g(\mathbf{z}_t, \mathbf{\Sigma}_0)}^{m_t=0} & \text{if } \mathbf{z}_t^H \mathbf{M} \mathbf{z}_t \leq c_0 \\ \overbrace{\alpha_1 g(\mathbf{z}, \mathbf{\Sigma}_1)}^{m_t=1} & \text{otherwise} \ , \end{cases} \tag{32}$$

where

$$\mathbf{z}_t^H \mathbf{M} \mathbf{z}_t = c_0 \tag{33}$$

represents the boundary equation when the two terms are equal, $\mathbf{M} = \mathbf{\Sigma}_0^{-1} - \mathbf{\Sigma}_1^{-1}$, and $c_0 = \ln(\frac{\alpha_0 |\mathbf{\Sigma}_1|}{\alpha_1 |\mathbf{\Sigma}_0|})$. Using this approach, the natural logarithm can be used to approximate the ML receiver as

$$\hat{\mathbf{X}}_{\text{ML}} \approx \arg\max_{\mathbf{X}} \sum_{t=1}^{T_b} \left[- \left| \mathbf{L}_{m_t}^H \mathbf{y}_t - \sqrt{\frac{E_s}{N_0 M_t}} \mathbf{L}_{m_t}^H \mathbf{H}\mathbf{x}_t \right|^2 + \ln\left(\frac{\alpha_{m_t}}{\pi^{M_r} |\mathbf{\Sigma}_{m_t}|}\right) \right] , \qquad (34)$$

where the noise state m_t can be determined at each time instant t by computing

$$m_t = \begin{cases} 0 & \text{if } (\mathbf{y}_t - \sqrt{\frac{E_s}{N_0 M_t}} \mathbf{H}\mathbf{x}_t)^H \mathbf{M} (\mathbf{y}_t - \sqrt{\frac{E_s}{N_0 M_t}} \mathbf{H}\mathbf{x}_t) \leq c_0 \\ 1 & \text{otherwise} , \end{cases} \qquad (35)$$

From (28) and (34), we note that the only difference between the MDR and the approximated ML receiver is the bias term $\ln(\frac{\alpha_{m_t}}{\pi^{M_r}|\mathbf{\Sigma}_{m_t}|})$, which depends on the noise state at the t^{th} time instant. It worths mentioning that the proposed suboptimum detector (34) is equivalent to a log-sum approximation [13], i.e., $\ln\left(\sum_m g_m(\cdot)\right) \approx \max_m \{\ln(g_m(\cdot))\}$. We note that the selection of the maximum term is equivalent to determining the noise state parameter m.

V. Simulation Results

In this section, we present a series of simulation results to validate our analysis by comparing the bit-error ratio (BER) of a conventional MDR, the ML receiver, and the approximated ML receiver (AMLR) in different impulse noise environments. In all cases, we consider a 2×2 Alamouti-OSTBC transmission scheme [14] of PSK signals over Rayleigh fading. Furthermore, we assume that the impulse noise statistics (A, Γ_j, and ρ_{jk}^m, $\forall j, k = 1, 2$) are known at the receiver. In the simulation of a multivariate MCA noise model, we consider only the first 10 terms, which are sufficient to approximate the full MCA density.

A. Uncorrelated MCA Noise

In this simulation, we consider a balanced MCA case, when the received interference has the same Gaussian factors at the receive antennas, i.e., $\Gamma_j = \Gamma$, $\forall j = 1, 2$. We assume three different sets of impulsive channels: near Gaussian noise ($A = 1,\ \Gamma = 10^{-1}$), a moderately impulsive channel ($A = 0.1,\ \Gamma = 10^{-2}$), and a strongly impulsive case ($A = 0.03,\ \Gamma = 10^{-3}$).

In Figure 1, we show the BER for the 2×2 MIMO system with different receivers over the considered impulsive channels. As we can see, the ML receiver and the proposed approximation offer substantial improvements over the conventional MDR. Although the proposed approximation is suboptimum, it is clear that its performance approaches the optimum one.

At high signal-to-noise ratios (SNRs), we note that the best provided performance of OSTBC can be achieved with Gaussian channels. The performance improvements decreases as the channel impulsiveness increases, i.e., $A \ll 1$. This is consistent with our analysis in (27), that for $M_r = 2$, the

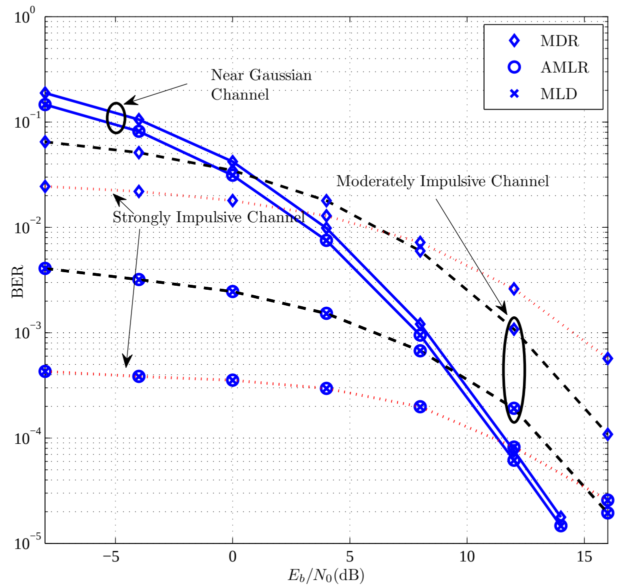

Fig. 1. Performance comparison of a 2×2 MIMO system with uncorrelated impulse noise

performance loss factor of the upper bound of PEPs is directly proportional to $(\frac{1}{A})^{M_t}$.

B. Correlated MCA Noise

Since the received interference often comes from common external sources to the receive antennas, recent studies showed that a significant level of noise correlation exists even when the antennas are far apart [3]. Here, we assume that the impulsive terms are arriving with the same correlation coefficients that differ from the Gaussian term correlation coefficient, i.e., $\rho_{jk}^m = \rho_{jk}^I$ for $m \geq 1$ and $\rho_{jk}^m = \rho_{jk}^G$ for $m = 0$.

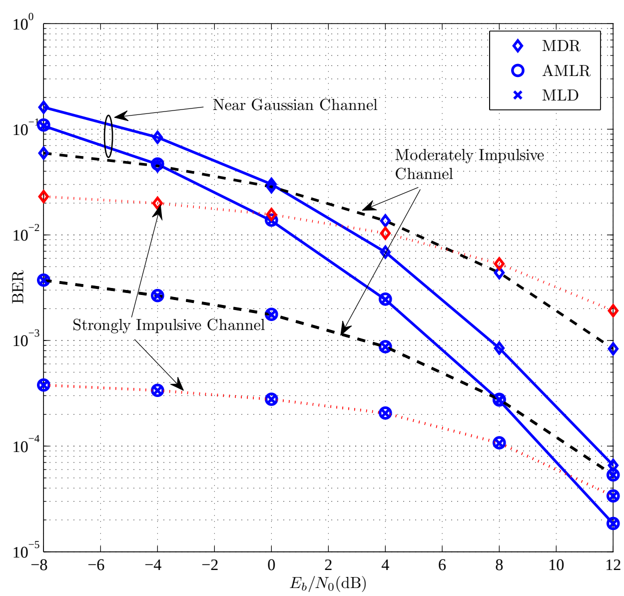

Fig. 2. Performance comparison of a 2×2 MIMO system with correlated impulse noise

To simulate this scenario, we use the same channel parameters of the uncorrelated case. We consider a situation where the Gaussian background noise has a less correlation, i.e., $\rho_{12}^{G} = 0.2$), than the impulsive events, i.e., $\rho_{12}^{I} = 0.75$. Figure 2 depicts the BER performance of this scenario. We note that the performance of a conventional MDR is much worse than its performance over uncorrelated channels, because it ignores the impulsive events and decorrelates only the Gaussian component with anyhow smaller correlation. Again at high SNRs, we note that the performance of OSTBC in impulse noise does not offer a good diversity gain as in nearly Gaussian noise, which confirms the analysis of Section IV.

Fig. 3 depicts the BER versus the impulse noise correlation ρ_{12}^{I} of the proposed AMLR with different levels of thermal noise correlation ρ_{12}^{G} (at SNR= 12 dB). The gap in the BER curves between the near Gaussian case and the moderately impulsive case represents the performance loss factor of OSTBC at the considered value of SNR. This gap is almost the same as the impulse noise correlation increases for all thermal noise correlation levels. This means that, the performance loss is slightly dependent on channel correlations. We further can note that the performance of AMLR is significantly improved as the impulse noise correlation increases, which was expected since the impulsive term distribution is the dominant term of the MCA density for both channels [11].

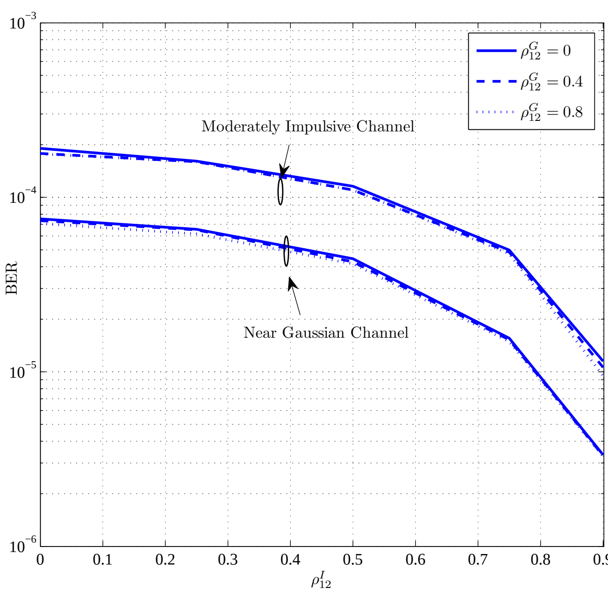

Fig. 3. Average BER versus noise correlation coefficients of a 2×2 MIMO system at $\mathrm{SNR} = 12$ dB

VI. Conclusion

For spatially correlated channels, the multivariate Class-A model can be used to model impulse noise at the receiver. This model can be seen as a conditional multivariate Gaussian distribution if the noise state is known. Based on this knowledge, the upper bound of pairwise error probabilities (PEPs) of orthogonal space time block coding (OSTBC) can be derived in a tractable form. In this analysis, we showed that the advantage of OSTBC over Gaussian noise is reduced in correlated impulse noise. Moreover, this limitation increases significantly with the numbers of transmit and receive antennas. Finally, we realized a simple approach to estimate the noise state at the receiver, which leads to an almost optimum detector.

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