

# Dynamic User Scheduling in Non-Orthogonal Multiuser MIMO-OFDM

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**Abstract**—We propose a multiuser scheduler based on signal-to-interference-and-noise ratio (SINR) constraints using a non-orthogonal space-division multiple access (SDMA) MIMO-OFDM. To mitigate the complexity needed for computing the linear transceiver filters with a possible number of non-feasible users, we propose to preselect the optimal users using a virtual signal-to-noise ratio (vSNR). This is computed assuming non-interfering users and a total power constraint. Solely depending on vSNR, we obtain thresholds that fulfill the equivalent SINR constraints for a different number of users. In this paper, vSNR thresholds are utilized to develop a simple multiuser scheduling routine. Finally, a performance comparison to the unscheduled case will be demonstrated.

**Index Terms**—MU-MIMO, OFDM, Non-orthogonal SDMA, SNR, SINR, Outage probability

## I. INTRODUCTION

Efficient multiuser (MU) transmission succeeds in minimizing the MU interference, which is dependent on two factors: the channel state information (CSI) and the number of users joining the same base station (BS). Assuming a priori knowledge of CSI at the transmitter, the power allocation across users and their transceiver filters need to be jointly adapted. In [3] the authors suggest to use an iterative approach that succeeds in diminishing the complexity compared to the convex optimization approach proposed in [4]. In this case, the sum mean-squared error (SMSE) between the transmitted and the received signals is iteratively minimized, i.e., for all data streams, with a total transmit power constraint. The algorithm in [3] has been modified to realize different QoS with SDMA in [1], however, without user selection. In [2], a greedy user selection is proposed using zero-forcing and dirty-paper (gZF-DP) with a non-linear combination, which shows quite high complexity.

In this paper, we study a non-orthogonal SDMA MU-MIMO downlink channel using OFDM to exploit both spatial and spectral domains for allocating

users. By using such a multiple access scheme, we are able to pre-estimate and assign the right users to the right subcarrier given the final individual SINR. However, to reduce the complexity of solving a non-orthogonal SDMA scheme with probably non-feasible users (i.e., some users may be not suitable for transmitting and we perform complex computations, iterations, and inappropriate space division among non-feasible users) we propose to reduce the number of users to the feasible ones, only. This can be easily achieved by considering the measured vSNR of each user assuming a virtual uplink channel similar to the channel setup in [1]. However, in this virtual uplink channel, the total power is divided equally among users, assuming that each user solely transmits to the BS in a dual uplink. Along with this, the SINR can be computed using an iterative approach, using the scaled gradient projection method, as in [5].

In order to analyze the system performance, we obtain the cumulative density function (CDF) curves for both vSNR and its equivalent SINR, i.e., illustrating the outage probabilities for different SINR/vSNR constraints [8]. We consider the vSNR to be an upper bound to the prospective SINR (relying on the outage analysis of vSNR) with a level of reliability. Nevertheless, this can be an indication to the corresponding SINR worst case scenario.

The rest of this paper is organized as follows: in Section II, our system model is introduced. In Section III, a description of the proposed user scheduling algorithm is presented. In Section IV, a sample result is depicted. Finally, in Section V, summaries and concludes this paper.

## II. SYSTEM MODEL

We consider a MU-MIMO downlink transmission scenario with, in total,  $N_T$  transmit antennas distributed at  $N_{BS}$  coordinated base stations (BSs),  $D$

user clusters where each cluster contains  $U$  non-cooperative users and each user is equipped with  $N_u$  receive antennas.  $N_R = \sum_{u=0}^{U-1} N_u$  represents the total number of receive antennas at all users within a certain cluster. The BS attempts to transmit  $M_u$  symbol streams to user  $u$ , where the sum of data streams is  $M = \sum_{u=0}^{U-1} M_u$ . Figure 1 shows four coordinated access points and a few mobile sets (MSs). These MSs are divided into two clusters, a high quality of service (QoS) cluster (QoS A) and a less quality set (QoS B); some of the MSs are not transmitting.

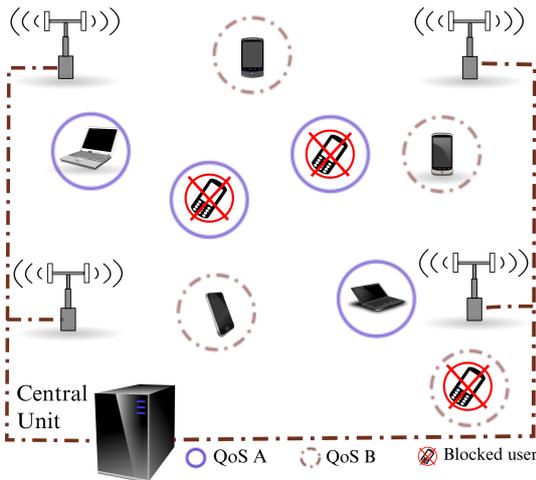


Fig. 1. Coordinated BSs with non-cooperative users with 2 QoSs, QoS A (low SER) and QoS B (high SER)

The frequency band is divided into  $N$  subcarriers using OFDM, i.e.,  $N$  MIMO channels in MIMO-OFDM. We assume a MIMO channel  $\mathbf{H}_{k,u} \in \mathcal{C} \mathcal{N}(0, \sigma_{\mathbf{H}}^2 \mathbf{I}_{N_u} \otimes \mathbf{I}_{N_T})$  as in [1], i.e.,  $\mathbf{H}_u \in \mathbb{C}^{N_u \times N_T}$ . This means that the channel matrix entries are uncorrelated zero mean circularly symmetric complex Gaussian (ZMCSCG) values and modeled as independent Rayleigh fading blocks with exponential decaying. Since we are targeting a down-link (DL) scenario, the users symbols are beamformed together at the BS using the following beamformer

$$\mathbf{F} = [\mathbf{F}_0 \mathbf{F}_1 \cdots \mathbf{F}_U], \quad (1)$$

where  $\mathbf{F}$  is the overall pre-coding matrix and  $\mathbf{F}_u \in \mathbb{C}^{N_T \times M_u}$ . Thus, multiplexing the base-band output at each MS  $u$ , we obtain (after receiver equalization)

$$\mathbf{W}_u \mathbf{r}_u = \mathbf{W}_u (\mathbf{H}_u \mathbf{F} \mathbf{x} + \mathbf{n}), \quad (2)$$

where  $\mathbf{x} = [\mathbf{x}_1^T \mathbf{x}_2^T \cdots \mathbf{x}_U^T]^T$  is the aggregated multiuser quadrature amplitude modulated symbol (QAM), such that  $\mathbf{x}_u \in \mathbb{C}^{M_u \times 1}$ .  $\mathbf{W}_u \in \mathbb{C}^{M_u \times N_u}$  is the user's individual post-processing matrix.  $\mathbf{n} \in$

$\mathbb{C}^{N_u \times 1}$  is the zero mean Gaussian noise vector with a variance  $\sigma_n^2$  per component.

In order to find the minimum sum-MSE, the following optimization problem must be solved

$$\begin{aligned} & \underset{\mathbf{F}, \mathbf{W}}{\text{minimize}} && \sum_{u=0}^{U-1} E_u^{\text{DL}} \\ & \text{subject to} && \text{Tr}(\mathbf{F}^H \mathbf{F}) \leq P_m, \end{aligned} \quad (3)$$

where  $E_u$  denotes the mean-squared error of the  $u^{\text{th}}$  user's symbols and  $P_m$  is the maximum transmit power. This optimization problem can be solved utilizing the MAC-BC duality using the same iterative gradient projection method proposed in [3] and modified in [1]. For simplicity, we assumed that the channel is perfectly known at the coordinated transmitters and at the individual users. Reader is referred to [1, Algorithm 1] for more details.

### III. MULTIUSER SCHEDULING BASED ON vSNR

In this paper, our aim is to assign users to the appropriate subcarriers and offer a fast solution to the complex Multi-User MIMO-OFDM system. This is achieved by classifying each subcarrier into one of the three cases presented below (based only on the measured received SNR values at each subcarrier) and decide for the appropriate users on each subcarrier:

- 1) fully loaded subcarriers - shared by all users,
- 2) partially used subcarriers - only the strong users are allowed to be multiplexed, and
- 3) unused subcarriers, which show very weak signals for all users or utilized by one of them, i.e., similar to the traditional frequency division multiple access (FDMA) technique.

#### A. Computing the vSNR

We propose to compute a virtual SNR (vSNR), which can be used to estimate the quality of each subcarrier instead of computing the actual SINR. This vSNR is equivalent to a single SNR, i.e., as if the BS is transmitting to a single user, however, scaled by the number of users  $U$ . Thus, we decompose the channel of each user (for each subcarrier) using eigenvalue decomposition as follows

$$\mathbf{H}_{k,u}^H \mathbf{H}_{k,u} = \mathbf{V}_{k,u} \mathbf{D}_{k,u} \mathbf{V}_{k,u}^H, \quad (4)$$

where  $k$  is the subcarrier index and  $\mathbf{D}_{k,u} = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{M_u})$  with  $\lambda_{k,l,u}$  as the eigenvalue of the  $l^{\text{th}}$  stream of the Hermitian matrix  $\mathbf{H}_{k,u}^H \mathbf{H}_{k,u}$ . Hence, the SNR of the  $u^{\text{th}}$  user's  $l^{\text{th}}$  stream is

$$\text{SNR}_{k,l,u} = \frac{\lambda_{k,l,u} p_{k,l,u}}{U \sigma_n^2}, \quad (5)$$

where  $U$  is the total number of users in each cluster and  $p_{k,l,u}$  is the power of the  $l^{\text{th}}$  stream. For simplicity, we assume that the power is allocated equally to each stream, i.e.,  $p_{k,l,u} = \frac{1}{M_u}$ .

To achieve the channel capacity, the authors in [6] showed that the multiuser throughput is easily maximized (assuming FDMA transmission) by maximizing the product  $\prod_{l=1}^{M_u} \lambda_{k,l,u}$ . In other words, select the user with the maximum geometric mean of the eigenvalues  $g_{u,k}$ , such that

$$g_{u,k} = \sqrt[M_u]{\prod_{l=1}^{M_u} \lambda_{k,l,u}}, \quad (6)$$

where  $M_u$  is the number of streams. The equivalent (virtual) SNR for every user  $u$  ( $\beta_u$ ) is given by

$$\beta_{k,u} = \frac{g_{u,k}}{M_u U \sigma_n^2}, \quad (7)$$

which is directly computed using the geometric mean in (6). Thus, it is necessary that all the users feedback their  $\beta_{k,u}$  to the BS. These  $\beta_{u,k} \forall u = 1..U$  have to be sorted in descending order. This sorting is repeated for the given  $N$  subcarriers. Hence, the user(s) with the strongest  $\beta_{k,u} \forall k = 1..N$  can only be scheduled for transmission using either FDMA or SDMA (simultaneous users share the same subcarrier). For each multiuser scenario, i.e.,  $\mu = 1, 2, \dots, U$ , we have to find the geometric mean of the vSNR for the  $\mu$  users as follows

$$\Upsilon_{k,\mu} = \sqrt[\mu]{\prod_{v=1}^{\mu} \overleftarrow{\beta}_{k,v}}, \quad \forall \mu = 1..U \ \& \ k = 1..N, \quad (8)$$

where  $\overleftarrow{\beta}_{k,v}$  are the users' vSNR sorted in descending order, i.e.,  $\overleftarrow{\beta}(1) > \overleftarrow{\beta}(2) > \dots > \overleftarrow{\beta}(U)$ . To compute the equivalent SINR $_{u,k}$ , similar to the computation of vSNR in (7) and (8), one has to find the eigenvalue of the Hermitian channel  $\mathbf{F}_u^H \mathbf{H}_u^H \mathbf{H}_u \mathbf{F}_u$ , i.e.,  $\hat{\lambda}_l$ , using eigenvalue decomposition. Hence, the equivalent SINR for each user ( $\nu_{k,u}$ ) is

$$\nu_{k,u} = \frac{\prod_{l=1}^{M_u} \hat{\lambda}_{l,i}}{\sum_{u=1, u \neq i}^U \prod_{l=1}^{M_u} \hat{\lambda}_{l,i} + \sigma_n^2} \quad (9)$$

and the geometric mean of the SINR ( $\Gamma_{k,\mu}$ ) for  $\mu$  users can be computed similar to (8) as follows

$$\Gamma_{k,\mu} = \sqrt[\mu]{\prod_{v=1}^{\mu} \overleftarrow{\nu}_{k,v}}, \quad \forall \mu = 1..U \ \& \ k = 1..N. \quad (10)$$

## B. Scheduler Design

In this paper, we design a sub-optimal SDMA multiuser scheduler. To simplify our computations, we proposed to set a few vSNR thresholds (vSNR $_T$ ) off-line, i.e., to forecast the quality of each individual subcarrier. To find the appropriate thresholds that guarantee specific QoSs, it is required to find the CDF curves of both  $\Upsilon$  (vSNR) and  $\Gamma$  (equivalent SINR) for all possible multiuser scenarios, i.e., for  $\mu = 1, 2, \dots, U$ . Based on these curves, we can easily find the appropriate percentage (1 – an outage probability  $P_T$ ) of the subcarriers which exceed a certain quality vSNR $_T$  (or its equivalent SINR $_T$ ). Hereby, we allow multiple users to share a percentage of  $1 - P_T$  of the subcarriers that exceeds the corresponding vSNR $_T$  on the CDF curves.

We divided the users into **five** different QoS groups with five different percentages: **10%** (the strongest 10% of the subcarriers are multiplexed or  $P_T = 20\%$ ), **30%**, **50%**, **70%**, and **80%** (80% of the subcarriers ( $P_T = 20\%$ ) can be multiplexed with higher SER and, however, higher multiplexing gain).

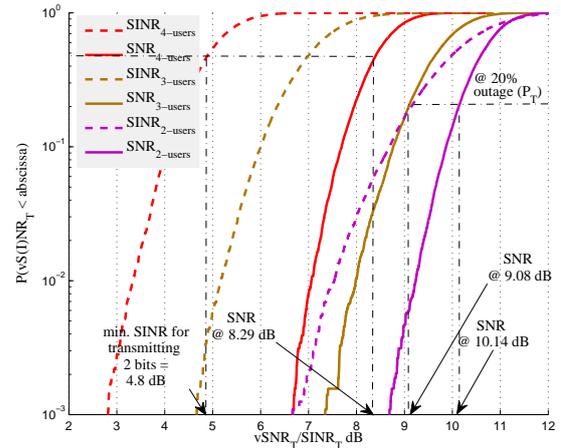


Fig. 2. Outage probability for different vSNR $_T$  and SINR $_T$  for 2, 3, and 4 users with a minimum SINR $_T = 4.8$  dB and a maximum outage probability  $P_T = 20\%$  (80% of the subcarriers can be multiplexed)

Fig. 2 depicts the CDF curves of both SINR and vSNR for different multiuser scenarios: 4, 3, and 2-user cases. The average signal to background white Gaussian noise ratio is set to 9 dB. The x-axis shows the different vSNR $_T$ /SINR $_T$  (the subcarriers quality); these are computed considering the geometric mean in (8) and (10). The outage probabilities  $P_T = 20\%$  (80% of the subcarriers exceed vSNR $_T$ ) works perfectly for the 3 and 2 case. At this outage,

the equivalent  $\text{SINR}_T$  (for the 3-user case) is 6 dB, which is enough to transmit 2 bits ( $\equiv 4.8$  dB). However, for 4 users at  $P_T = 20\%$ , the minimum  $\text{SINR}_T$  (4.8 dB) is not achieved, i.e., transmitting 2 bits is not feasible. However, the 4.8 dB rather results in an outage of 50% (transmitting only on the strongest 50% of the subcarriers), which means a  $\text{vSNR} \geq 8.29$  dB. Decreasing the noise variance will shift these curves to the right. Table I lists the  $\text{vSNR}$  threshold equivalent to the outage probabilities 20%, 50%, and 90%. This is listed for  $\mu = 4, 3$ , and 2 users at an average SNR of 9 dB and 15 dB.

TABLE I  
OUTAGE PROBABILITIES AND  $\text{vSNR}$  THRESHOLDS IN DB

$1/\sigma_n^2$	9 dB			15 dB		
Outage	20%	50%	90%	20%	50%	90%
4 users	8.29	8.49	9.33	16.91	17.49	18.33
3 users	9.08	9.73	10.74	18.08	18.77	19.73
2 users	10.14	10.72	11.48	19.14	19.69	20.52

In the following, we discuss the complete scheduling algorithm.

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**Algorithm 1** Multiuser scheduling using  $\text{vSNR}$

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**Initialize:** the maximum number of users  $U$ , the scheduling table  $\mathbf{S} = \mathbf{0}^{N \times 1}$ , and compute  $g_{u,k}$  for all users

**Input:**  $b_m$  bit/stream and Table I with all the  $\text{vSNR}_\mu$

**Output:** the sorted indices  $\mathbf{S}$

- 1: compute  $\Upsilon_{k,\mu} \forall k = 1..K$  for each individual scenario, i.e.,  $\mu = 1 \dots U$ , where  $\mu$  is the number of users allowed to be multiplexed
  - 2:  $\mu \leftarrow 1$
  - 3: **repeat**
  - 4:   **repeat**
  - 5:     **if**  $\Upsilon_{k,\mu} \geq \text{vSNR}_\mu$  **then**
  - 6:        $\mathbf{S}(k) \leftarrow \mu$
  - 7:     **end if**
  - 8:      $\mu \leftarrow \mu + 1$
  - 9:   **until**  $\mu > U$
  - 10:  $k \leftarrow k + 1$
  - 11: **until**  $k > N$
  - 12: allocate  $b_m$  bits/stream to all users in  $\mathbf{S}$
- 

#### IV. SIMULATION RESULTS AND ANALYSIS

In our simulation, we model a MU MIMO-OFDM transmission system with parameters similar to that in the LTE Advanced [9]. The total number of subcarriers is 1024, where 5 clusters of 128 subcarriers

are dedicated for 5 groups of users. The remaining subcarriers can be used for signaling, pilots, and other services. Four access points are assumed with 8 antennas in total. The number of the receive antennas at each MS is assumed to be 2. For simplicity, let the number of streams dedicated for each user,  $M_u$ , be equal to the number of antennas per user  $N_u$ , i.e., 2 as well.

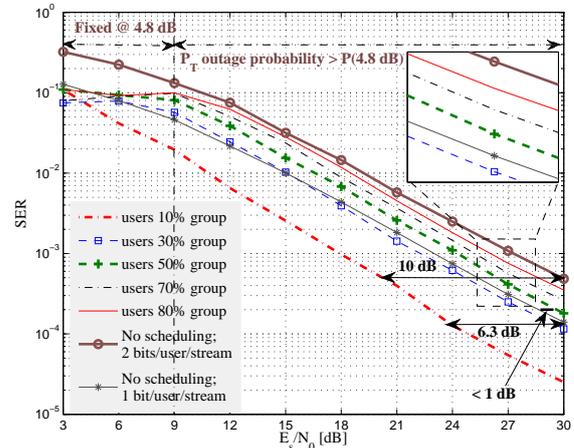


Fig. 3. SER for 10%, 30%, 50%, 70%, and 80% QoS groups. The region below 9 dB shows the performance when the 20% outage probability is not sufficient for transmitting 2 bits.

Figure 3 depicts the symbol-error ratio (SER) for the different multiuser groups, i.e.,  $1 - P_T = 10\%, 30\%, 50\%, 70\%$ , and  $80\%$ . It is clear that the group of users which is allowed to be simultaneously multiplex over 10% of the given subcarriers are performing the best. It achieves **10 dB** better error performance than the unscheduled approach with **2 bits/stream/user**. However, from Fig. 4, we can see that the 10% group achieves only 1.1 bits/stream/user (on average). Nevertheless, it is still outperforming the unscheduled scheme, with 1 bit/stream/user, by **6.3 dB** (Fig. 3).

Another interesting result in Fig. 3 is the performance of the 50% group; it is only 1 dB less than the unscheduled scheme with **1 bit/stream/user**. However, from Fig. 4, it gains **63%** more throughput compared to that of the unscheduled scheme with 1 bits/stream/user, i.e.,  $> 1.63$  bits/stream/user. It is also important to mention that the 70% group outperforms the unscheduled scheme with **2 bits/stream/user** by more than **2.7 dB** (at  $5 \times 10^{-4}$  in Fig. 3) and loses only 0.23 bits compared to the same case (Fig. 4).

Finally, Fig. 5 shows the subcarriers utilization of the different scheduling schemes. It is clear from

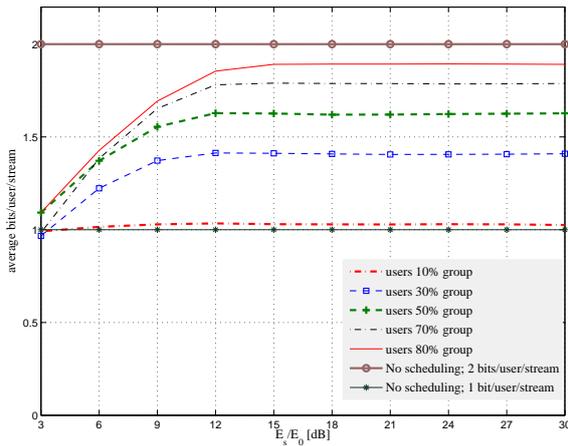


Fig. 4. Number of bits/user/stream for the 5-QoS groups with a comparison to the unscheduled cases using 1 and 2 bits/user/stream

this figure that the 4-user case is utilizing 80% of the subcarriers when the vSNR threshold are set to the values corresponds to the outage probability of 20%. The utilization of the 4-user case falls beyond 10% at the 90% outage probability. However, at an outage probability of, e.g., 90%, the utilization of the 3-user and the 2-user cases are much higher than the designed 10%. The reason is that these 2 or 3 users are the ones with the strongest vSNR. In this case, the multiuser diversity is exploited more than in the 4 user case. It is even more utilized in the 2-user case than in the 3-user one.

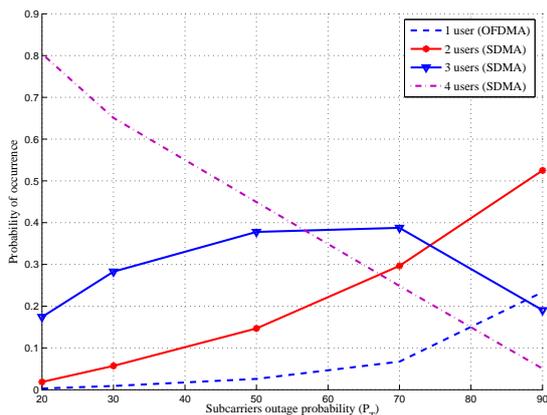


Fig. 5. Subcarriers utilization for different QoS groups

## V. CONCLUSION

Instead of selecting the users after computing their optimum beamformers, we opt for a simpler mul-

tiuser scheduler. Hereto, we forecast the quality of each individual subcarrier and the maximum number of users it can accommodate using the simple virtual SNR computation. This makes our solution fast; even faster than the unscheduled multiuser MIMO-OFDM. This is due to minimizing the number of users needed to be multiplexed simultaneously and computing their optimum beamformers.

Additionally, we succeed in devoting different QoSs by just changing the number of subcarriers utilized for multiuser transmission. Furthermore, utilizing only 50% of the subcarriers for multiuser SDMA transmission results in 63% more throughput compared to the unscheduled scheme with 1 bit/user/stream.

## VI. ACKNOWLEDGMENT

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