Prioritized Adaptive Modulation for MIMO-OFDM Using Pre-Ordered SIC

Khaled Hassan and Werner Henkel
School of Engineering and Science, Jacobs University, Transmission Systems Group (TrSyS)
28759 Bremen, Germany, Email:{k.hassan & w.henkel}@jacobs-university.de

Abstract—In MIMO transmission, channel state information (CSI) is crucial for achieving channel adaptation. However, the inaccuracy of CSI may induce severe interferences. Hence, limitations of linear equalizers to combat severe interference and noise enhancements necessitate the need for investigating non-linear equalization schemes. Thus, we propose a modified successive interference cancellation (SIC) technique based on the well-known V-BLAST non-linear spatial equalizer. First, we implement a linear pre-processing filter in order to achieve presorting of the channel at the transmitter assuming severe CSI errors. This simplifies the complexity of our non-linear equalizer significantly by implementing the ordering module at the transmitter. Additionally, an unequal-error protection (UEP) bit-loading algorithm is proposed to keep the strongest layers of symbols well protected against errors. This reduces the error propagation inside our SIC process. Finally, a comparison to an MMSE linear equalization shows that our design operates at a lower symbol-error ratio (SER) with almost identical complexity.

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) channels can be easily decomposed into non-interfering eigenchannels (also known in literature as eigenbeams [1]) with different gains. These orthogonal eigenchannels can be used as an underlying structure for transmitting adaptive modulation with different priorities. Therefore, it is necessary to design techniques that allocate the existing spatial resources and adapt the modulation scheme such that the overall performance satisfies a required QoS. However, in frequency-selective channels, MIMO performance deteriorates, seriously. This can effectively be resolved by sub-dividing the bandwidth into smaller subcarriers, such that the channel appears to be flat within these narrow subcarriers. Thus, multicarrier techniques, e.g., OFDM, are used in conjunction with the existing MIMO systems to transfer such frequency selective channels into $N$ narrower flat sub-bands.

The MIMO-OFDM combination takes advantage of simple frequency domain equalization and channel adaptation capabilities along the spectral domain. Generally, this is achievable using closed-loop transmission (with a feedback from the receiver to the transmitter) and utilizing the individual subcarriers and the orthogonal eigenchannels. Accordingly, the bit rates and powers can be adapted to the channel variations. To achieve optimum performance, the complete channel state information (CSI) would need to be known accurately at the transmitter. However, a perfect CSI knowledge is indeed a rather impractical assumption due to estimation errors, limited feedback conditions, channel feedback delays, and/or quantization errors. Hence, a partial CSI at the transmitter is a more realistic assumption between the two extremes, perfect CSI [2] and no-CSI [3].

There exist many partial CSI schemes in which we select two main models: the channel quantized/delayed feedback $H$ [3], [4] and the channel covariance feedback $R_H$ [5]. Since the decomposition of the instantaneous channel (using singular value decomposition (SVD)) varies from that of the delayed or inaccurate CSI, the transmission orthogonalization cannot be guaranteed any more. Thus, CSI inaccuracy induces an inter-eigen interference (IEI), which results in performance deterioration. Hence, to compensate for the orthogonality distortion, we consider implementing different pre- and post-processing units at the transmitter and the receiver, respectively [2]. We first propose to use a linear post-processing, i.e., a linear spatial equalizer using minimum-mean square error (MMSE) criterion. However, the limitations of linear equalizers to combat severe interference and noise enhancements (due to some weak channels) necessitate the need for non-linear equalization [6]. Thus, we propose to use a successive interference cancellation technique based on the well-known V-BLAST non-linear spatial equalizer [7].

Based on the algorithms proposed in [3], we implement a modified UEP bit-loading algorithm which is capable of adapting MIMO-OFDM. Thus, bits and power are allocated along the different subcarriers and their eigenchannels, i.e., considering space-and-frequency bit-loading. We also realize UEP by fulfilling arbitrary performance margin separations between the given protection classes. In our simulation model, we investigated both the perfect and the imperfect CSI discussed in [3]–[5], [8].

The rest of this paper is organized as follows. Section 2 discusses our proposed channel model and the required precoding. Section 3 discusses the linear and non-linear spatial equalization. Section 4 describes our UEP adaptive modulation. Section 5 discusses the results. Finally, our findings are concluded in the last section.

II. MIMO CHANNEL MODEL

We consider a MIMO-OFDM system deploying $N_T$ transmit antennas (with $N_T$ IFFTs), $N_R$ receiver antennas (with $N_R$ FFTs), and $N$ subcarriers. In order to model the two dimensions of the MIMO-OFDM (space and frequency) which are used in our UEP bit-loading, we consider a MIMO channel...
matrix $\mathbf{H}_k \in \mathbb{C}^{N_R \times N_T}$ for each subcarrier $k$ [3]. For rich-scattering environments, the elements of $\mathbf{H}_k$ can be considered as i.i.d. with a zero mean circularly symmetric complex Gaussian (ZMCSGC) distribution [2]. Thus, the normalized channel (scaled to the number of transmit antennas $N_T$) is given by $\text{vec}(\mathbf{H}_k) \sim \mathcal{CN}(0, \sigma_{\mathbf{H}}^2 \mathbf{I}_{N_R N_T})$, where the rich-scattering assumption guarantees to have a diagonal covariance matrix $\sigma_{\mathbf{H}}^2 \mathbf{I}$. In limited scattering environments, the resultant covariance matrix is not diagonal any more. Thus, $\text{vec}(\mathbf{H}_k) \sim \mathcal{CN}(0_k, \mathbf{R}_n)$. In order to simplify the channel correlation parameters, we follow the separable Kronecker correlation model discussed in [9], which can easily separate between the correlations present near the transmitter and the correlations at the receiver. Herewith, the channel covariance matrix is

$$\mathbf{R}_n = \mathbb{E}\{\mathbf{H}_k \mathbf{H}_k^H\} \otimes \mathbb{E}\{\mathbf{H}_k^H \mathbf{H}_k\} = \mathbf{R}_R \otimes \mathbf{R}_T.$$  

(1)

For mathematical convenience, we assume no correlation at the receiver side, i.e., $\mathbf{R}_R \approx \mathbf{I}$. Consequently, the MIMO channel matrix in [9] is reduced to

$$\mathbf{H}_k = \mathbf{H}_k \mathbf{R}_T^{1/2},$$  

(2)

where the elements of $\mathbf{H}_k$ are assumed to be i.i.d. with ZMCSGC distribution and $\mathbf{R}_T \in \mathbb{C}^{N_T \times N_T}$ is the transmit covariance matrix [9], which is given by

$$\mathbf{R}_T = \mathbb{E}\{\mathbf{H}_k^H \mathbf{H}_k\} = \mathbf{H}_k^H \mathbf{R}_T \mathbf{H}_k.$$  

(3)

where $\mathbb{E}\{\mathbf{H}_k^H \mathbf{H}_k\} = \mathbf{I}$. Therefore, estimating the transmit antenna correlation $\mathbf{R}_T$ is sufficient to envisage link adaptation based on spatial correlations [5]. Hence, the eigenvectors of $\mathbf{R}_T$ are used as a beamforming matrix which directs the transmission to the direction of the highest channel gain.

### A. Eigen-beamforming based on quantized/delayed CSI

Our erroneous CSI model is stated as $\mathbf{H}_k = \mathbf{H}_k + \mathbf{Z}_k$ [3], where $\mathbf{H}_k$ is the delayed/quantized channel and the estimated CSI at the receiver (assuming a perfect CSI at the receiver side). $\text{vec}(\mathbf{Z}_k) \sim \mathcal{CN}(0, \sigma_{\mathbf{Z}}^2 \mathbf{I})$ represents the CSI error. Hence, the received vector $\mathbf{y}_k$ of the $k$th subcarrier, after applying the precoding matrix $\mathbf{V}_k$ (of the instantaneous channel $\mathbf{H}$) and the power loading matrix $\mathbf{P}_k^{1/2}$ (similar to [3]), can be written as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{V}_k \mathbf{P}_k^{1/2} \mathbf{x}_k + \mathbf{n}_k = \mathbf{U}_k \mathbf{D}_k^{1/2} \mathbf{V}_k \mathbf{P}_k^{1/2} \mathbf{x}_k + \mathbf{n}_k,$$

(4)

where, using an SVD, $\mathbf{H}_k = \mathbf{U}_k \mathbf{D}_k^{1/2} \mathbf{V}_k^H$, $\mathbf{D}$ is a diagonal matrix of the eigenvalues $\lambda_k$ (of $\mathbf{H}_k^H \mathbf{H}_k$), $\mathbf{x}_k$ is the transmitted vector and $\mathbf{n}_k \sim \mathcal{CN}(0, \sigma_{\mathbf{Z}}^2 \mathbf{I}_{N_T})$ is an additive white Gaussian noise vector and $\mathbf{V}_k$ is a unitary matrix resulting from $\mathbf{V}_k = \mathbf{D}_k^{-1/2} \mathbf{V}_k^H$. $\mathbf{Psi}_k$ represents the aggregated channel matrix and $\mathbf{P}_k^{1/2}$ is a diagonal matrix containing the allocated power values, which is already part of $\mathbf{Psi}_k$. Under perfect CSI conditions, i.e., $\sigma_{\mathbf{Z}}^2 = 0$ and $\mathbf{V}_k = \mathbf{I}_{N_T}$, we obtain

$$\mathbf{Psi}_k = \mathbf{U}_k \Lambda_k,$$

(5)

where $\mathbf{U}_k$ is a unitary matrix and $\Lambda_k$ is the diagonal matrix $\mathbf{D}_k^{1/2} \mathbf{P}_k^{1/2}$.

**Lemma 1:** Without loss of generality, if $\Lambda_k$ is a sorted (descending) diagonal matrix and $\mathbf{U}_k$ is a unitary matrix; therefore, the Frobenius-norm square of the of the first column of $\mathbf{Psi}_k$ is its maximum eigenvalue and the first raw of $\mathbf{Psi}_k^{-1}$ is its minimum eigenvalue.

In the imperfect CSI case, $\mathbf{V}_k$ is a general (non-diagonal) matrix and thus

$$\mathbf{Psi}_k = \mathbf{U}_k \mathbf{D}_k^{1/2} \mathbf{V}_k \mathbf{P}_k^{1/2}.$$  

(6)

However, $\mathbf{D}_k^{1/2} \mathbf{V}_k \mathbf{P}_k^{1/2}$ is a strong diagonal dominant if the CSI error is small. Even more, we found that the order of the channel strength follows exactly the order of the channel eigenvalues. Thus, for ordered eigenchannels (from high to low), the first column of $\mathbf{Psi}_k$ has the highest gain, while the last column has the least strength. In other words, the squared vector norm of $\mathbf{Psi}_k(:,1)$ has the highest value. Hence, if a non-linear SIC is considered at the receiver, no further effort is need for reordering the receiver matrix $\mathbf{W}$ as the received channel columns are already sorted in the correct sequence (from high to low).

### B. Channel Covariance Eigen-beamforming

Similar to [5], the eigen-beamforming based on the channel covariance feedback is used to maximize the received power along the eigenvectors of the transmit correlation matrix. Hereeto, the beamforming is realized by pre-multiplying the transmit symbols by the unitary pre-processing matrix $\mathbf{U}_T$, which results from decomposing $\mathbf{R}_T$, i.e., $\mathbf{R}_T = \mathbf{U}_T \mathbf{D}_T \mathbf{U}_T^H$. where $\mathbf{U}_T$ is a unitary matrix containing $N_T$ eigenvectors (which is used as an eigen-beamforming) and $\mathbf{D}_T$ is a diagonal matrix of the eigenvalues of $\mathbf{R}_T$.

### III. EQUALIZATION FOR DIAGONALIZED MIMO

The post-combiner at the receiver side is designed to be a spatial equalizer in order to mitigate the IEI caused by the MIMO channels. For imperfect CSI, the diagonalization cannot be guaranteed. We first consider a linear MMSE to combat the noise enhancement, however, without further reduction of the noise superposition that results due to the residual IEI. In order to reduce the remaining IEI in the resulting channel $\mathbf{Psi}$, we propose to use a non-linear successive interference cancellation by modifying the V-BLAST algorithm which was first introduced in [7].

#### A. Minimum-mean Square Error Linear Equalizer

The minimum-mean square error linear equalizer (L-MMSE) matrix (in [10]) is given by

$$\mathbf{W} = \{(\mathbf{Psi}^H \mathbf{Psi} + \sigma_{\mathbf{Z}}^2 \mathbf{I})^{-1} \} \mathbf{Psi}^H.$$  

(7)

At high SNR, both L-MMSE and zero-forcing (ZF) equalizers are performing very similar. Thus, both equalizers suffer from the same non-avoidable cross-talks if the channel is
not perfectly known at the transmitter, i.e., due to CSI errors \( \mathbf{V}_k \neq \mathbf{I}_{N_T} \). Hence, it is necessary to eliminate the remaining interference using successive interference reduction methods, i.e., non-linear equalization.

**B. Successive Interference Cancellation using V-BLAST**

The well-known V-BLAST is mainly a successive interference cancellation (SIC) receiver. This technique consists of data being transmitted from different transmit antennas onto the space forming different layers [7]. These layers are successively decoded at the receiver, where the interference is successively canceled. Certainly, the order of detecting the symbols affects the overall performance. Thus, our modified V-BLAST utilizes the precoded (and also preordered) overall channel \( \mathbf{\Psi} \) to avoid the iterative search-and-sort process used in the original algorithm in [7]. In this case, the first column of \( \mathbf{W} = (\mathbf{\Psi}^H \mathbf{\Psi} + \mathbf{\sigma}^2_n \mathbf{I})^{-1} \mathbf{\Psi}^H \) represents the smallest received MSE while its last column is the highest one. However, for very high CSI errors, we may need to perform iterative search-and-sort steps similar to the original algorithm in [7]. Assume a small CSI error variance \( \mathbf{\sigma}^2_m \), the 3 main steps to perform SIC for a preordered received channel are:

1. Consider \( \mathbf{y}(1) \) to be the strongest symbol; then decode using the appropriate MMSE (column of \( \mathbf{W} \)).
2. Interference is canceled using the previously regenerates symbol (after hard decision).
3. The remaining interference is nulled from each symbol using MMSE linear equalizer.

Here we consider implementing V-BLAST on the resulting channel \( \mathbf{\Psi}_k \) after bit-loading, power allocation, and beamforming for imperfect CSI. Thus, the bits are allocated assuming UEP bit-loading using sorted subcarriers, i.e., the eigenchannels are sorted according to their carrier gain-to-noise ratios (CGNR), where we place the most important data on the higher eigenchannel (UEP bit-loading is discussed in the next section). Therefore, the prioritizes transmission guarantees that the first received symbol already enjoys very few errors.

Additionally, the detection of the strongest symbol is carried out without considering these few errors due to IEI as stated in the V-BLAST algorithm [7]. Later, the detected symbol is used to cancel the remaining interference on the weaker eigenchannels. In Algorithm 1, we present our modified V-BLAST algorithm assuming a presorted channel matrix \( \mathbf{\Psi}_k \) for all subcarriers \( k \):

Algorithm 1 V-BLAST for preordered \( \mathbf{\Psi}_k \) ∀ \( k = 1..N \)

**Initialize**: \( i \leftarrow 0 \) (the row with the minimum MSE), \( k_i \leftarrow i \), and \( \mathbf{W} = (\mathbf{\Psi}^H \mathbf{\Psi} + \mathbf{\sigma}^2_n \mathbf{I})^{-1} \mathbf{\Psi}^H \)

1. **repeat**
   2. if the normalized CSI error variance is not too high, e.g., \( \mathbf{\sigma}^2_m < 0.25 \); see Appendix A then
   3. using LEMMA 1; set the equalizer minimum MSE row to the iteration index, i.e., \( k_i = i \)
   4. else
   5. if \( \mathbf{\sigma}^2_m \geq 0.25 \) (see Appendix A) or if covariance feedback case:
   6. find the minimum \( k_i = \arg\min_m \| \mathbf{W}(m,:) \|^2 \)
   7. **end if**
   8. compute the MMSE (nulling) row: \( \mathbf{w}_{k_i} = \mathbf{W}(k_i,:) \)
   9. implement the MMSE interference nulling: \( \mathbf{\hat{x}}_{k_i} = \mathbf{w}_{k_i} \mathbf{y}_i \), where \( \mathbf{y}_i \) has less interference
   10. detect the symbol: \( \mathbf{\hat{x}}_{k_i} = \text{detect}(\mathbf{\hat{x}}_{k_i}) \)
   11. perform the cancellation of the detected component: \( \mathbf{y}_{i+1} = \mathbf{y}_i - \mathbf{\Psi}_i(1, k_i) \mathbf{\hat{x}}_{k_i} \)
   12. if the normalized CSI error variance is high enough, e.g., \( \mathbf{\sigma}^2_m \geq 0.3 \), then
   13. set the columns \( k_i \) of \( \mathbf{\Psi}_i \) to all zeros, i.e., for not selecting it again in Line 6
   14. compute the new equalize matrix for the \( i \)th iteration: \( \mathbf{\Psi}_i = (\mathbf{\Psi}_i^H \mathbf{\Psi}_i + \mathbf{\sigma}^2_n \mathbf{I})^{-1} \mathbf{\Psi}_i^H \)
   15. **end if**
   16. go to the next column: \( i \leftarrow i + 1 \)
   17. **until** \( i = R \) (maximum MIMO channel rank)

**IV. UEP Adaptive Modulation**

In order to adapt the MIMO-OFDM transmission, we require, at least, the partial channel matrix \( \mathbf{H}_k \) or the transmit covariance matrix \( \mathbf{R}_T \). Herewith, the \( R \) eigenbeams for all \( N \) subcarriers, i.e., \( RN \) eigenchannels, are used to allocate bits and power. Thus, to realize different UEP classes, hypothetical thresholds are found to divide the sorted eigenchannels in order to allocate \( N_R \) classes with different margin separation \( \Delta \gamma_j \). In this paper, we consider the intuitive subcarrier sorting mechanisms proposed in [4]. In this case, the subcarriers with the highest eigenbeams are allocated to the important data. For comparison, we consider the robust sorting which allocates the least important data to the subcarriers with the highest CGNR, i.e., to avoid allocating the very weak subcarriers [4].

In order to proceed with our algorithm, all the \( RN \) eigenchannels have to be combined in a long buffer \( \mathcal{M} \in \mathbb{Z}^{1 \times NR} \). The sorting procedure has to go sequentially through this buffer in order to satisfy the two-dimensional sorting (see Figure 1). Furthermore, \( \tau_j \) are set within this buffer such that the UEP requirements (the classes bit rates \( T_j \) and the margin separations \( \Delta \gamma_j \)) are fulfilled by modifying these thresholds, thereby changing the number of subcarriers in each class. However, the limited/partial channel feedbacks are expected to result in performance degradations. These limitations are described extensively using the SER performance in our
transmission. Note: we proceed with the bit-loading schemes for MIMO-OFDM results and analysis section assuming different scenarios. Now, we proceed with the bit-loading schemes for MIMO-OFDM transmission. Note: m is the middle class.

V. RESULTS

In our simulation, we model a 4 × 4 MIMO-OFDM transmission system with N = 512 subcarriers for each eigenbeam. Thus, the maximum permissible power emission from the transmitter P_T is normalized to unity. Additionally, we assumed a maximum target rate of 3072 bits to be transmitted over 3 classes, i.e., each gets 1024 bits. All the MIMO channels are assumed to undergo a Rayleigh fading channel with 9 equally spaced delayed paths with an exponential decaying power delay profile [5]. Here, we assume 2 different CSI models: 1) quantized/delayed CSI with an error variance σ^2_B = 25% of the original channel variance; 2) channel correlation feedback similar to the first model in [5].

In Fig. 2, we depict the performance of a linear MMSE equalizer versus the performance of the adaptive V-BLAST proposed in Section III-B. For intuitive sorting with 4-D, i.e., R = 4, eigen-beamforming, our proposed adaptive V-BLAST outperforms the MMSE by almost 12 dB at an SER of 10^-5. Additionally, the V-BLAST is only 5 dB worse than the perfect CSI using 4-D eigen-beamforming, i.e., comparing the results of the perfect intuitive sorting mechanism with 4 dimensional (4-D) beamforming to the V-BLAST results, at an SER of 10^-6. Since the V-BLAST receives the symbols sorted according to their channel gains, the algorithm starts with the least priority data in case of robust sorting, which has the highest SER. Therefore, error propagates to the highest priority data resulting in the error floor seen at high-SNR.

Figure 3 depicts a 2-D, i.e., R = 2, eigen-beamforming using a linear MMSE equalizer and a non-linear VBLAST with MMSE nulling receiver. As expected, our adaptive V-BLAST using intuitive sorting outperforms the equivalent linear MMSE spatial equalizer and performs very close to the perfect CSI conditions with a loss of 1.8 dB. The reason for this significant improvement is that we started with the most protected symbol, i.e., fewer errors to propagate.

Figure 4 depicts the performance of the channel covariance feedback with adaptive MIMO-OFDM using the same algorithm^1 in [5]. We show that the adaptive V-BLAST with intuitive sorting performs much better than the equivalent intuitive sorting using a linear MMSE equalizer (as in Fig. 4).

\(^1\)similar to Algorithm 2, however, without spectral dependencies
The order of the classes is kept safe due to utilizing the channel correlation matrix eigenvalues and eigenvectors. However, the separations become narrower, as the error between the instantaneous channel and the channel covariance is relatively high, which threatens the performance of V-BLAST original algorithm. Finally, we can see that the adaptive V-BLAST results in a performance gain (slightly more than 7.4 dB) compared to the non-adaptive V-BLAST, even with channel correlation feedback. Thus, it can be thought of as a practical solution for adaptive wireless MIMO systems with partial CSI, especially due to its very low computational complexity.

VI. CONCLUSIONS

We showed that selected margin separations between the data classes are achieved, even under partial CSI. Our modified adaptive V-BLAST succeeds in mitigating the inter-eigen interference by successive cancellation. For quantized/delayed CSI, with relatively low CSI errors, the V-BLAST ordering is achieved by following, directly, the same order of the semi-diagonalized channel. This enhances the performance, and reduces the sorting and searching effort. For channel correlation feedback, the V-BLAST also succeed in mitigating interference, even under a sub-optimum channel ordering. The only issue to consider here, is the error floor seen at high SNR using the “robust” sorting. This is mainly due to the error propagation from the least important classes to the higher ones.

APPENDIX A

Figure 5 shows the probability of error for receiving the order of the four eigenbeams of a 4×4 MIMO channel. It is very clear that the first eigenbeam (and the strongest one) is detected at the first column with a very high probability (only with 18% error at $\sigma^2_{\text{error}} = 25\%$). Increasing $\sigma^2_{\text{error}}$ may lead to a change in the order of the presorted matrix, i.e., may require sorting at the receiver. However, this will be only a fraction of the 3 dB gain of the sorting. Additionally, the first layer is always protected by mean of UEP allocation.

REFERENCES