Least-Squares Iterative Peak-to-Average Ratio Reduction for MIMO-OFDM Systems

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***Abstract*—This manuscript addresses peak-to-average ratio (PAR) reduction in orthogonal frequency division multiplexing (OFDM) based multiple-input multiple-output (MIMO) systems. A new technique for PAR reduction in point-to-point scenarios is proposed. Singular value decomposition (SVD) algorithms usually sort the singular values of a MIMO channel in descending order. The last singular value and the associated eigenchannels of a MIMO-OFDM channel are often very weak. Not using them for data transmission will offer redundancy for PAR reduction. These eigenchannels are used to approximate the peaks which exceed a given target value in a least-squares fashion. This approximated exceedence model is then subtracted from the original signal in time domain for PAR reduction. It has been shown that a remarkable gain can be obtained with the proposed algorithm with a negligible increase in the average power and capacity loss.**

I. Introduction

MIMO-OFDM is a promising technique which has gained quite some popularity in modern day communications technology. However, a major drawback of OFDM systems is the high peak-to-average ratio (PAR) which drives the power amplifiers (PA) to operate in its non-linear range. In order to make PAs more power efficient, different techniques have been proposed for PAR reduction. An overview of the proposed techniques for PAR reduction can be found in [1], [2]. The well-known techniques for PAR reduction in multi-antenna systems are Selected Mapping (SLM) [3]–[6], Partial Transmit Sequences (PTS) [7], [8], and Tone Reservation (TR) [11]–[13].

In this paper, we propose a new method for PAR reduction in point-to-point (PtP) MIMO-OFDM systems using the weakest eigenchannel(s) to estimate the threshold excursion caused by the remaining dimensions, which are then subtracted from the original signal in time domain for PAR reduction. The procedure is iterated to reach the PAR target value.

The rest of the paper is structured as follows. In Section II, we present the key idea. The system model is described in Section III. Section IV gives a detailed description of the core idea of estimating the exceedence values above a certain target value. In Section V, we present the results and Section VI finally concludes the paper.

II. Key idea

For a point-to-point MIMO-OFDM system with perfect channel state information (CSI), the channel matrix $\mathbf{H}(n)$ ($M_r \times M_t$, with $M_r = M_t$) in DFT domain at carrier n can be diagonalized using SVD, as

$$\mathbf{H}(n) = \mathbf{U}(n) \cdot \mathbf{\Lambda}(n) \cdot \mathbf{V}^H(n) \ , \tag{1}$$

where $\mathbf{U}(n)$ and $\mathbf{V}(n)$ are unitary matrices such that $\mathbf{U}^H \cdot \mathbf{U} = \mathbf{V}^H \cdot \mathbf{V} = \mathbf{I}$ and $\mathbf{\Lambda}(n)$ is a diagonal matrix of the singular values of $\mathbf{H}(n)$, i.e.,

$$\mathbf{\Lambda}(n) = \begin{pmatrix} \sigma_{1,1}(n) & 0 & 0 & 0 \\ 0 & \sigma_{2,2}(n) & 0 & 0 \\ \vdots & \dots & \ddots & \vdots \\ 0 & 0 & 0 & \sigma_{M_r,M_t}(n) \end{pmatrix} . \tag{2}$$

The singular values are usually sorted in a descending fashion by SVD algorithms, i.e., $\sigma_{1,1} > \sigma_{2,2} > \dots > \sigma_{M_r,M_t}$. The last singular value σ_{M_r,M_t} is often very weak such that the associated eigenchannels are hardly suited for data transmission. Thus, not using them would offer redundancy for peak-to-average ratio (PAR) reduction without a lot of cost in data rate. Throughout this paper, it is assumed that the last eigenchannel is too weak and is hence utilized for PAR reduction.

III. System model and pre-coding

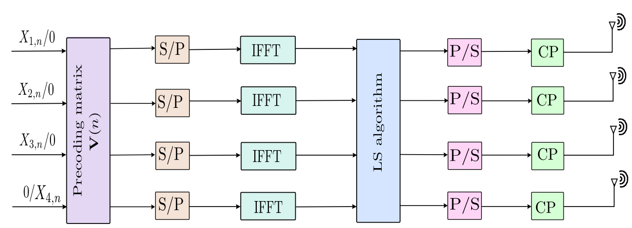

Fig. 1. Transmitter system model of MIMO-OFDM for LS algorithm

Figure 1 shows a transmitter model of a MIMO-OFDM system with basic blocks. Let M_t be the total number of transmit and M_r be the total number of receive antennas. Let $\mathbf{X}(n) = (X_{1,n}, X_{2,n}, \cdots, X_{M_t,n})^T$, where $X_{\mu,n}$ is the nth input symbol at the μth ($\mu = 1, 2, \cdots, M_t$) spatial channel, be the input data vector. Let $\mathbf{Y}(n) = (Y_{1,n}, Y_{2,n}, \cdots, Y_{M_r,n})^T$ be the output vector, where $Y_{\rho,n}$ is the output symbol of the ρth ($\rho = 1, 2, \cdots, M_r$) spatial channel. For a better understanding, we use the input-output relation in frequency domain. The input data vector $\mathbf{X}(n)$ is precoded by $\mathbf{V}(n)$ at the transmitter i.e., $\tilde{\mathbf{X}}(n) = \mathbf{V}(n) \cdot \mathbf{X}(n)$, whereas the signal at the receiver is post-multiplied by $\mathbf{U}^H(n)$ to obtain the output $\mathbf{Y}(n)$, i.e., $\mathbf{Y}(n) = \mathbf{U}^H(n) \cdot \tilde{\mathbf{Y}}(n)$, as shown in Fig. 2. Mathematically,

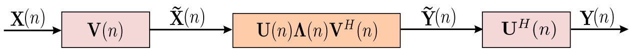

Fig. 2. SVD MIMO diagonalization

$$\mathbf{Y}(n) = \overbrace{\mathbf{U}(n)^H \cdot \mathbf{U}(n)}^{\mathbf{I}} \cdot \underbrace{\mathbf{\Lambda}(n) \cdot \overbrace{\mathbf{V}^H(n) \cdot \mathbf{V}(n)}^{\mathbf{I}}}_{\mathbf{H}(n)} \cdot \underbrace{\mathbf{V}(n) \cdot \mathbf{X}(n)}_{\tilde{\mathbf{X}}(n)} \quad (3)$$

$$\mathbf{Y}(n) = \mathbf{\Lambda}(n) \cdot \mathbf{X}(n) \, . \quad (4)$$

For the $M_t \times M_r = 4 \times 4$ case, Eq. (4) in matrix form is rephrased as

$$\mathbf{Y}(n) = \begin{pmatrix} \sigma_{1,1}(n) & 0 & 0 & 0 \\ 0 & \sigma_{2,2}(n) & 0 & 0 \\ 0 & 0 & \sigma_{3,3}(n) & 0 \\ 0 & 0 & 0 & \sigma_{4,4}(n) \end{pmatrix} \begin{pmatrix} X_{1,n} \\ X_{2,n} \\ X_{3,n} \\ X_{4,n} \end{pmatrix} . \quad (5)$$

We have assumed that the last singular value is very weak and is reserved to offer redundancy for PAR reduction. Let $\mathbf{S}(n)$ denote the first dimensions used for data transmission, i.e.,

$$\mathbf{S}(n) \;= \begin{pmatrix} X_{1,n} & X_{2,n} & X_{3,n} & 0 \end{pmatrix}^T ,$$

Let $\mathbf{R}(n)$ represents the reserved spatial dimension, which will be used to estimate and model the peak values above a certain threshold, i.e.,

$$\mathbf{R}(n) \;= \begin{pmatrix} 0 & 0 & 0 & X_{4,n} \end{pmatrix}^T ,$$

whereas $X_{4,n}$ is the corrective signal that is used for PAR reduction. As shown in Fig. 1, the input data vector $\mathbf{S}(n)$ and the corrective signal $\mathbf{R}(n)$ are pre-processed as

$$\tilde{\mathbf{S}}(n) = \mathbf{V}(n) \cdot \mathbf{S}(n) = \mathbf{V}(n) \cdot \begin{pmatrix} X_{1,n} & X_{2,n} & X_{3,n} & 0 \end{pmatrix}^T \quad (6)$$

and

$$\tilde{\mathbf{R}}(n) = \mathbf{V}(n) \cdot \mathbf{R}(n) = \mathbf{V}(n) \cdot \begin{pmatrix} 0 & 0 & 0 & X_{4,n} \end{pmatrix}^T . \quad (7)$$

The two are then transformed into time domain and are subtracted, i.e.,

$$\tilde{\mathbf{x}} = \tilde{\mathbf{s}} - \tilde{\mathbf{r}} = \mathbf{F}^{-1}\mathbf{V}\mathbf{X} = \mathbf{F}^{-1}\mathbf{V}(\mathbf{S} - \mathbf{R}) \, , \quad (8)$$

using a block-IDFT matrix $\mathbf{F}^{-1}$ with blocks of diagonal submatrices with $M_t = M_r (= 4)$ identical diagonal elements $w_{n,k} = \frac{1}{\sqrt{N}} e^{j2\pi kn/N}$. All vectors $\mathbf{X}, \tilde{\mathbf{X}}, ...$ are obtained by concatenating the vectors $\mathbf{X}(n), \tilde{\mathbf{X}}(n), ...$ Now, the goal is to estimate and model the exceedence values by $\mathbf{R}$ in DFT domain to finally subtract the model.

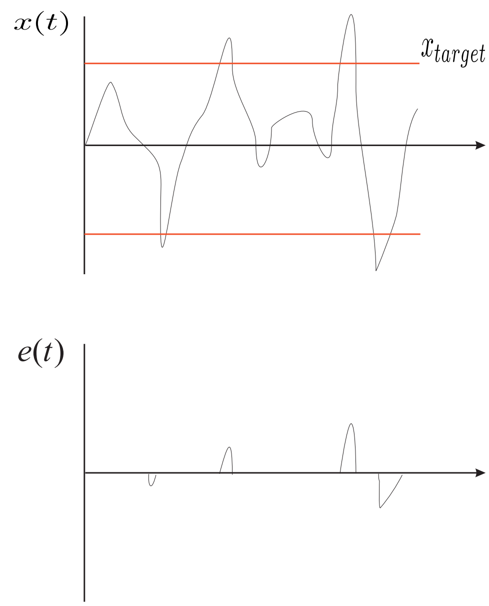

Fig. 3. Representation of exceedence values

IV. Modeling the Peak Excursions by the Last Spatial Dimension

To estimate and model the peak excursion of the transmitted signal $\tilde{\mathbf{s}}$ (with no data on the reserved eigenchannels), the algorithm searches for any peak value that exceeds a PAR target value. Let $\mathbf{e}$ be a vector representing the exceeding excursions in time domain, i.e.,

$$\mathbf{e} = ((e_{1,1}e_{2,1}e_{3,1}e_{4,1}), \cdots, (e_{1,n}e_{2,n}e_{3,n}e_{4,n}))^T , \quad (9)$$

where

$$e_{\mu,k} = \begin{cases} 0 & \text{for } x_{\mu,k} \leq x_{target} \, , \\ x_{\mu,k} - e^{j\angle(x_{\mu,k})} \cdot x_{target} & \text{for } x_{\mu,k} > x_{target} \, . \end{cases} , \quad (10)$$

where, $x_{\mu,k}$ is the kth sample at the μth spatial dimension. Let $\mathbf{E}$ be the DFT-domain counterpart of $\mathbf{e}$, i.e., $\mathbf{E} = \mathbf{F} \cdot \mathbf{e}$, where $\mathbf{F}$ is a block-DFT matrix. We will use two approaches to estimate the peak values exceeding the given threshold, which are actually equivalent.

1) 1st approach: In order to estimate and model the exceedence excursions by the reserved spatial dimension, Eq. (7) can be reformulated as

$$\begin{pmatrix} \mathbf{V}(0) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}(1) & \dots & \mathbf{0} \\ \vdots & \dots & \ddots & \vdots \\ \mathbf{0} & \dots & \dots & \mathbf{V}(N-1) \end{pmatrix} \begin{pmatrix} \vec{\mathbf{0}} \\ X_{4,0} \\ \vdots \\ \vec{\mathbf{0}} \\ X_{4,N-1} \end{pmatrix} = \boldsymbol{\varphi} \, , \quad (11)$$

where $\mathbf{V}(n)$ is the nth precoding unitary matrix, $\mathbf{0}$ is a 4×4 zero matrix and $\vec{\mathbf{0}} = (0 \quad 0 \quad 0)^T$, which in more compact form can be written as

$$\boldsymbol{\varphi} = \mathbf{V} \cdot \mathbf{R} \, , \quad (12)$$

where $\mathbf{V}$ is a block-diagonal unitary matrix. The column vector $\boldsymbol{\varphi}$ shall approximate $\mathbf{E}$ in a least-squares sense at all frequencies representing the determined exceedence peaks, i.e.,

$$\min_{\mathbf{R}} ||\boldsymbol{\varphi} - \mathbf{E}||^2 \ . \tag{13}$$

Equation (13) can be solved for $\mathbf{R}$ as a least-squares estimate,

$$\mathbf{R} = \mathbf{V^{-1}} \cdot \mathbf{E} \ . \tag{14}$$

Inside $\mathbf{R}$, only the components at 4th row, i.e., $(4,:)(i)$, $i = 1, ..., N$ will be kept, eliminating (zeroing) all others. The modified $\mathbf{R}$ is then pre-coded, transformed into time domain using the IFFT, and is subtracted from the original signal for PAR reduction.

2) 2nd approach: As shown, Eq. (7) essentially only addresses the last columns of the blocks $\mathbf{V}(i)$ inside the $\mathbf{V}$ matrix. Thus, Eq. (7) is rephrased as

$$\begin{pmatrix} \mathbf{V}_{:4}(0) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{:4}(1) & \dots & \mathbf{0} \\ \vdots & \dots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_{:4}(N-1) \end{pmatrix} \begin{pmatrix} X_{4,0} \\ X_{4,1} \\ \vdots \\ X_{4,N-1} \end{pmatrix} = \boldsymbol{\varphi} \ , \tag{15}$$

where $\mathbf{V}_{:4}(n)$ represents the 4th column (: stands for all rows) of the nth $\mathbf{V}$ matrix, which can be represented in a compact form as

$$\boldsymbol{\varphi} = \mathbf{V}_{:4} \cdot \mathbf{R}_4 \ , \tag{16}$$

where $\mathbf{V}_{:4}$ is a block diagonal matrix with the 4th column of $\mathbf{V}(n)$ at the nth diagonal, and $\mathbf{R}_4$ contains only the last spatial components. In a least-squares sense, the column vector $\boldsymbol{\varphi}$ shall approximate $\mathbf{E}$ according to

$$\min_{\mathbf{R}_4} ||\boldsymbol{\varphi} - \mathbf{E}||^2 \ . \tag{17}$$

In Eq. (17), the number of equations are more than the number of unknowns. This leads to a pseudo-inverse solution. Solving Eq. (17) for $\mathbf{R}_4$ in least-squares sense results in

$$\mathbf{R}_4 = (\mathbf{V}_{:4}^H \cdot \mathbf{V}_{:4})^{-1} \cdot \mathbf{V}_{:4}^H \cdot \mathbf{E} \ , \tag{18}$$

where H stands for Hermitian (conjugate transpose). Both approaches are equivalent. The first one, however, is the less complex, since no pseudo-inverse needs to be computed. The procedure is iterated, as the peaks exceeding the target value are not estimated exactly by the least-squares approach in one iteration. The least squares algorithm is described as follows.

Least Squares algorithm

1) Initialize $\mathbf{S}$ to be the DFT-domain information vector, with the reserved dimension set to zero. Precode it using the pre-processing matrix as shown in Eq. 6.
2) Initialize the time-domain solution $\tilde{\mathbf{s}}$, i.e., $\tilde{\mathbf{s}} = \text{IFFT}(\tilde{\mathbf{S}})$. Set $i = 0$.
3) Initialize $\mathbf{e}$ to represent the exceedence excursion according to Eq. (10) and transform it into DFT domain, i.e., $\mathbf{E} = \text{FFT}(\mathbf{e})$.
4) Approximate $\mathbf{E}$ by the last spatial dimension using equations (14) or (18). In case of using the 1st approach (Eq. 14), set the first three dimensions to zero. Precode it using the preprocessing matrix $\mathbf{V}$ and transform it into time domain applying the IFFT modulator, i.e., $\tilde{\mathbf{r}} = \mathbf{F^{-1}}\tilde{\mathbf{R}}$.
5) Update the time-domain vector
$$\tilde{\mathbf{s}}^{(i+1)} = \tilde{\mathbf{s}}^{(i)} - \tilde{\mathbf{r}} \ , \tag{19}$$
$i = i + 1$ and go to Step 3.

i is the iteration counter, and $\tilde{\mathbf{s}}$ denotes the $M_t \cdot N$ long time-domain vector, where M_t is the number of transmit antennas and N is the IFFT length. The structure follows (9).

One should note that it is very unlikely for reasonable antenna numbers (no massive MIMO) that more than one antenna channel will see a value exceeding the threshold at the same time. Hence, the least-squares approach will not approximate the peak, but yield a result reduced by a factor of M, since it tries, at the same time, to approximate the zeros (no exceedences) in the other antenna channels.[1] This is the drawback of an l_2 norm instead of l_∞. For an optimum performance of the algorithm, we introduce a weighting factor γ in the algorithm, i.e., $\gamma = M$. The weighting can be realized by modifying steps 3 or 5 of the presented LS algorithm. Modifying Step 3 means weighting the peak limit excursions before mapping them onto the last spatial dimension. Equation (10) becomes

$$e_{\mu,k} = \begin{cases} 0 & \text{for } x_{\mu,k} \leq x_{target} \ , \\ \gamma(x_{\mu,k} - e^{j\angle(x_{\mu,k})} \cdot x_{target}) & \text{for } x_{\mu,k} > x_{target} \ . \end{cases} \tag{20}$$

Alternatively, one could instead write Step 5, Eq. (19) as

$$\tilde{\mathbf{s}}^{(i+1)} = \tilde{\mathbf{s}}^{(i)} - \gamma\tilde{\mathbf{r}} \ . \tag{21}$$

V. Results and Discussion

For a MIMO-OFDM system with transmitter-sided precoding, the average power is distributed over all spatial dimensions. Moreover, we add a time-domain signal $\tilde{\mathbf{r}}$ to the original signal $\tilde{\mathbf{s}}$ for PAR reduction, thus, the average power is slightly increased with every iteration. The PAR after applying the algorithm is defined in here as

$$\text{PAR} = \frac{\max_{\forall_\mu,\forall_k} |x_{\mu,k} + \gamma r_{\mu,k}|^2}{\sigma^2} \ , \tag{22}$$

where k is the sample index, and $\sigma^2 = E_{\forall_\mu,\forall_k}\{|x_{\mu,k}|^2\}$ is chosen to be the average power (averaged over all spatial dimensions) without any PAR measures.

The channel matrix is chosen as in [3], however, with a channel length of $l_h = 20$. For simulation, we chose a 4×4 MIMO-OFDM system with 128 carriers and a 16-QAM modulation. To check and evaluate the performance, the complementary cumulative distribution function (ccdf) is considered, which is the probability that the current PAR exceeds a certain threshold τ_{th}, i.e., $Pr\{\text{PAR} > \tau_{th}\}$ [3].

Figure 4 shows the simulation results for a target PAR value of 5.0 dB. A gain of approximately 6.4 dB is obtained at 10^{-5} with 25 iterations. It is also clear from Fig. 4 that a sufficient gain is already obtained with 5-10 iterations (approx. 3.8 - 4.8 dB, respectively).

[1] Illustration: Assume e and $M-1$ zeros to be approximated in least-squares sense. This means $\frac{d}{dx}[(x-e)^2 + (M-1)(x-0)^2] = 0 \Longrightarrow x = e/M$.

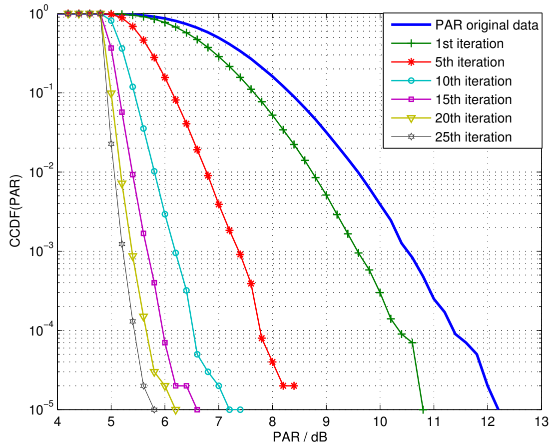

Fig. 4. CCDF(PAR) of the LS algorithm for a PAR target value of 5.0 dB without mean power constraints ($\gamma = 1$)

The complexity of the algorithm is minimized by weighting the correction signal used for PAR reduction. Figure 5 shows simulation results for *weighted LS* with a weighting factor $\gamma = 4$ and a PAR target value of 5 dB. As shown in Fig. 5, a gain of 6.4 dB is obtained at 10^{-5} with as few as 5 iterations. As can be seen from the figure, a gain of approximately 4 dB is obtained with the first iteration and as much as 5.0 dB with only two iterations.

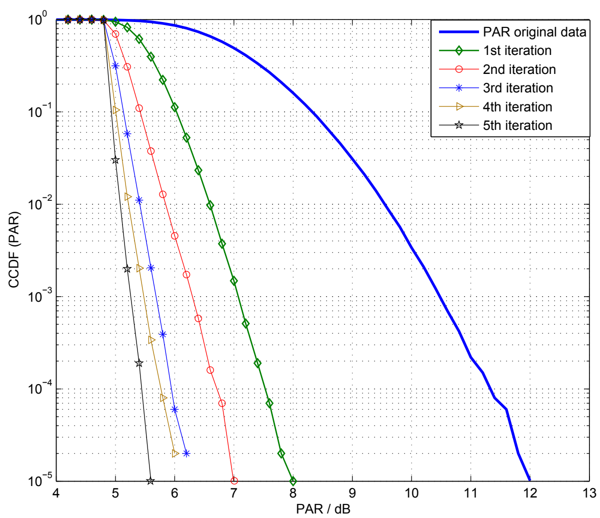

Fig. 5. CCDF(PAR) of the weighteed LS algorithm for a PAR target value of 5.0 dB without mean power constraints ($\gamma = 4$)

As mentioned earlier, for modeling the exceedence excursions, our algorithm searches for the peak values that exceeds a given target value on all spatial dimensions in time domain. These values are transformed into DFT domain, and are estimated by the last spatial dimension. The estimated model is re-transformed into time domain and are subtracted from the original signal for PAR reduction. However, in doing so, two facts needs to be taken into consideration for the LS algorithm:

1) Capacity loss: Loss in channel capacity due to the reserved eigenchannels.
2) Relative mean power increase ΔE: Increase in the average power per iteration due to addition of the corrective signal, and performance of the algorithm with mean power constraint.

A. *Capacity analysis of the proposed algorithm*

The proposed algorithm reserves the weakest eigenchannels, thus, it is necessary to analyze the channel capacities for a MIMO-OFDM channel with and without the LS algorithm. The capacity of a MIMO channel is equal to the sum of the capacities of the independent parallel channels, i.e.,

$$C_{MIMO} = \sum_{i=1}^{\min(M_r, M_t)} \log_2(1 + \alpha_i \sigma_i^2) \, , \tag{23}$$

where σ_i^2 is the ith singular value of the channel matrix $\mathbf{H}$, $\min(M_r, M_t)$ is the minimum of the number of receive or transmit antennas, and $\alpha_i = \Phi_i/\zeta^2$ denotes the transmitter-sided signal-to-noise ratio (SNR) of the ith parallel subchannel. For a MIMO-OFDM channel, (23) is extended to

$$C_T = \sum_{n=1}^{N} \sum_{i=1}^{\min(M_r, M_t)} \log_2(1 + \alpha_i(n)\sigma_i^2(n)) \, , \tag{24}$$

where $\sigma_i^2(n)$ and $\alpha_i(n)$ are the singular values and the SNRs at the nth subcarrier, respectively.
For our LS algorithm, (24) becomes

$$C_{LS} = \sum_{n=1}^{N} \sum_{i=1}^{\min(M_r, M_t)-1} \log_2(1 + \alpha_i(n)\sigma_i^2(n)) \, , \tag{25}$$

omitting the weakest (reserved) singular values.
For tone reservation (TR) [11], the sum is over $N - \theta$ subcarriers, where θ are the reserved tones. For the TR algorithm, Eq. (24) is rephrased as

$$C_{TR} = \sum_{\substack{N-\theta \\ \text{carriers}}} \sum_{i=1}^{\min(M_r, M_t)} \log_2(1 + \alpha_i(n)\sigma_i^2(n)) \, . \tag{26}$$

Figure 6 shows the capacity curves for a 4×4 MIMO-OFDM channel with and without reserved eigenchannels and for 10 % reserved tones. It is obvious from Fig. 6 that the loss in the channel capacity for the LS algorithm is negligible at low SNR and almost very little at high SNR values. Figure 6 also shows that the capacity loss due to the proposed algorithm is smaller than that of 10 % tone reservation.
It is assumed that the transmitter has perfect channel state information, thus, the power can be optimally allocated to the parallel subchannels. The channel capacity in terms of the power allocation $\Phi_i(n)$ to the ith parallel channel at the nth subcarrier is given as [14]

$$C_T = \max_{\Phi_i : \sum_i \Phi_i \le \Phi} \sum_{n=1}^{N} \sum_{i=1}^{\min(M_r, M_t)} \log_2\left(1 + \frac{\Phi_i(n)\sigma_i^2(n)}{\zeta^2}\right) . \tag{27}$$

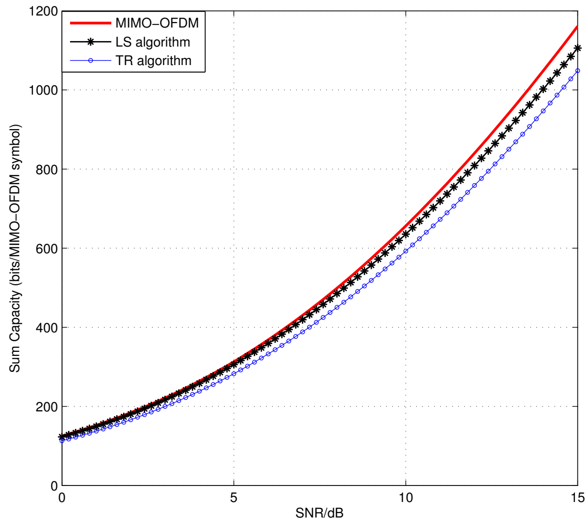

Fig. 6. Capacity curve of a 4×4 MIMO-OFDM channel with and without reserved eigenchannels, 10 % TR, and 128 carriers, averaged over 10^5 channel models

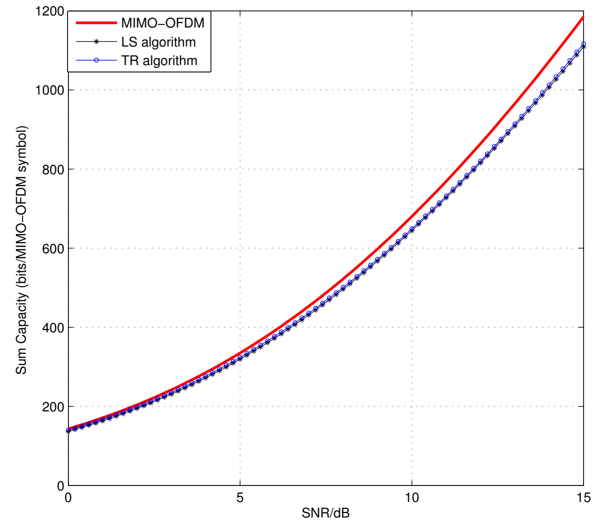

Fig. 7. Capacity curve of a 4×4 MIMO-OFDM channel with and without reserved eigenchannels, using water filling, 10 % TR, and 128 subcarriers, averaged over 10^5 channel models

For the LS algorithm, omitting the last spatial dimension leads to

$$C_{LS} = \max_{\Phi_i : \sum_i \Phi_i \leq \Phi} \sum_{n=1}^{N} \sum_{i=1}^{\min(M_r,M_t)-1} \log_2 \left(1 + \frac{\Phi_i(n)\sigma_i^2(n)}{\zeta^2}\right) , \quad (28)$$

and for the TR algorithm with no data transmission on $N - \theta$ subcarriers, Eq. (27) can be rephrased as

$$C_{TR} = \max_{\Phi_i : \sum_i \Phi_i \leq \Phi} \sum_{\substack{N-\theta \\ \text{carriers}}} \sum_{i=1}^{\min(M_r,M_t)} \log_2 \left(1 + \frac{\Phi_i(n)\sigma_i^2(n)}{\zeta^2}\right) . \quad (29)$$

Using an SVD [15], the MIMO channel is decomposed into parallel single-input single-output (SISO) channels. The optimization solution then leads to a water-filling power allocation for the MIMO-OFDM channel as,

$$\Phi_i(n) = \left(\beta - \frac{\zeta^2}{\sigma_i^2(n)}\right)^+ , \quad (30)$$

where β is the water level chosen such that

$$\sum_{n=1}^{N} \sum_{i=1}^{\min(M_r,M_t)} \Phi_i(n) = \Phi ,$$

$\Phi_i(n)$ is the power at the ith eigenmode and nth bin of the channel, and $(x)^+$ is defined as $\max(x, 0)$.
Figure 7 shows the capacity curves for a 4×4 MIMO-OFDM channel with and without reserved eigenchannels using water filling. The capacity curves are similar to the ones in Fig. 6, with negligible capacity loss at low SNR values and very little loss in capacity at high SNR values, however, the capacity gap is somewhat widened for water filling at higher SNR values.

B. Mean power effects of the proposed algorithm

The relative mean transmit power increase ΔE, due to addition of the corrective signal $\tilde{\mathbf{r}}$ to the transmitted signal $\tilde{\mathbf{s}}$, is defined as

$$\Delta \mathrm{E} = 10 \log_{10} \frac{\mathrm{E}\{||\tilde{\mathbf{s}}^i + \tilde{\mathbf{r}}^i||_2^2\}}{\mathrm{E}\{||\tilde{\mathbf{s}}||_2^2\}} , \quad (31)$$

where $\mathrm{E}\{||\tilde{\mathbf{s}}||_2^2\} = \sigma^2$ is the average power of the transmitted signal without any PAR measures and $\mathrm{E}\{||\tilde{\mathbf{s}}^i + \tilde{\mathbf{r}}^i||_2^2\}$ is the average power after the ith iteration.
Figure 8 shows the simulation results for different ΔE. As visible from Fig. 8, for a 0.1 dB increase in the average transmit power, the gain is approximately 4.2 dB, which is an almost negligible increase in the transmit power, and a gain of almost 6.2 dB is reached for a marginal increase in the mean power of ΔE = 0.25 dB.
Mean power effects of weighted LS: The simulation results for weighted LS with different mean power constraints are shown in Fig. 9. A gain of approximately 4 dB is obtained with a 0.1 dB increase in the average power which is almost negligible and almost 6.2 dB are reached for a marginal increase in the mean power of ΔE = 0.3 dB.

VI. Conclusions

We proposed a weighted least-squares iterative approach for peak-to-average ratio reduction in a point-to-point MIMO-OFDM system. Peak values exceeding a given target value are modeled by the weakest eigenchannels. Simulation results show that a gain of approximately 6.4 dB is obtained with as few as 5 iterations at a CCDF of 10^{-5}. This gain is obtained at the expense of a minor

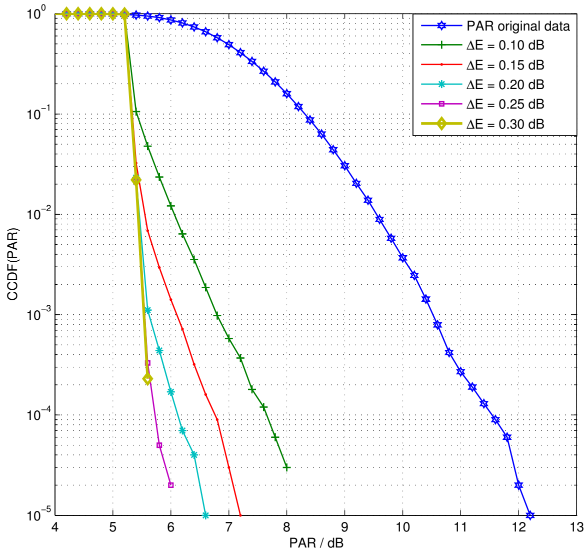

Fig. 8. CCDF(PAR) of the LS algorithm for a PAR target value 5.0 dB with different mean power constraints ΔE, number of iterations = 8

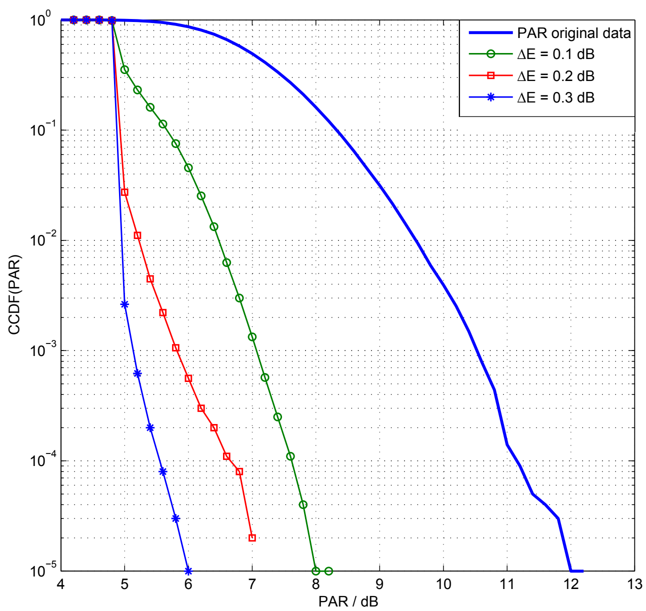

Fig. 9. CCDF(PAR) of the weighted LS algorithm with a weighting factor γ = 4.0 and PAR target value 5.0 dB for different mean power constraints ΔE, number of iterations = 8

increase in average power and a slight loss in channel capacity.

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